Christ the King Catholic Primary School

Calculation Progression Document

(Revised 2023)

Contents

Introduction

Christ the King's Calculation Progression Policy, is based upon guidance from 5 sources:

i) NCETM

ii) White Rose calculations document

iii) The National Curriculum and appendix

iv) Development Matters 2021

v) Mastering Numbers project

It was reviewed and revised in 2021.

For all calculation skills and concepts, it is important that children experience concrete, visual and abstract representations **alongside** each other, in order to strengthen their understanding of the concept.

Application

Calculation skills should be applied to:

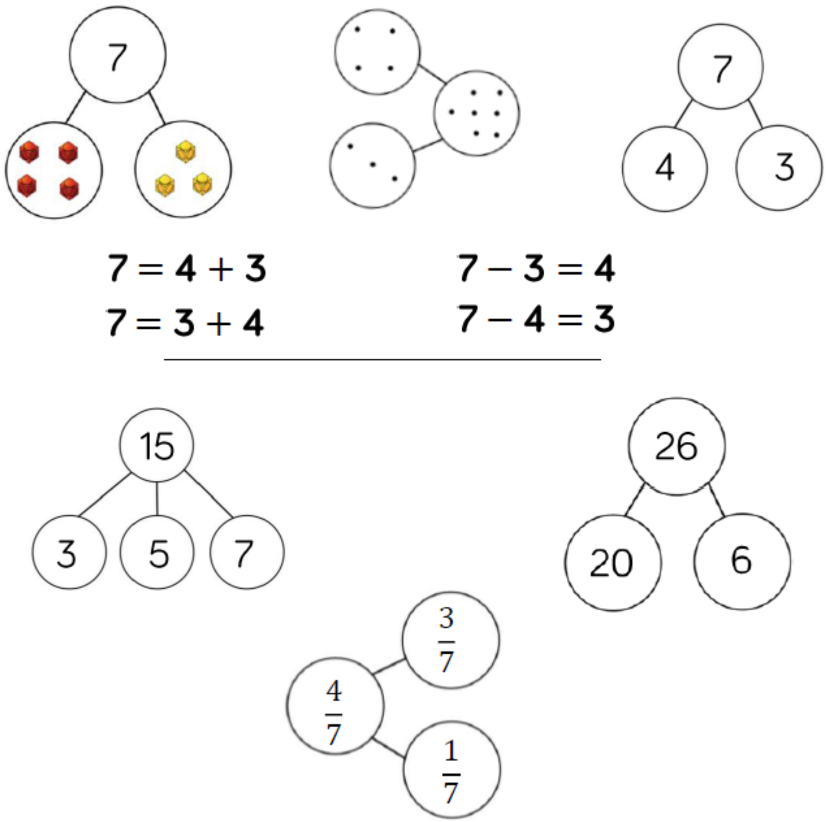
- inverse operations
- missing box questions, including the missing box representing the operation
- measures, including money and time
- word problems and reasoning problems
- open ended investigations

NOTE: It is also important to use the = sign in various places and that children have a true understanding of its meaning.

Representations

At CTK, we follow the White Rose schemes of learning as a basis for our teaching. The following are representations we use throughout the school. (from White Rose Calculations Document)

Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

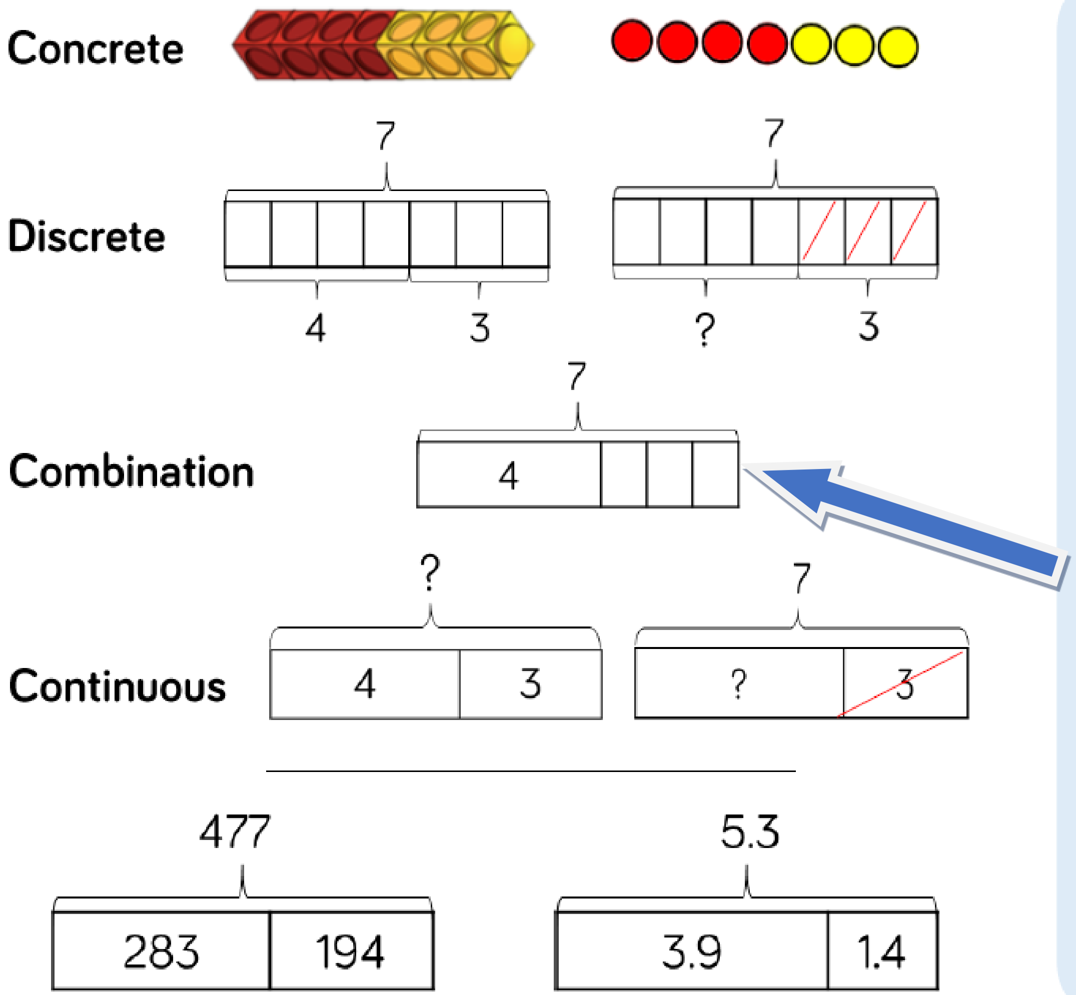
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

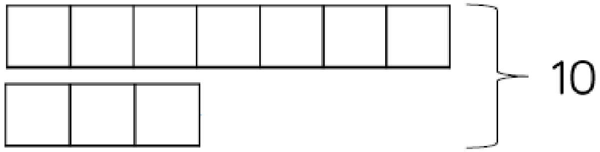
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

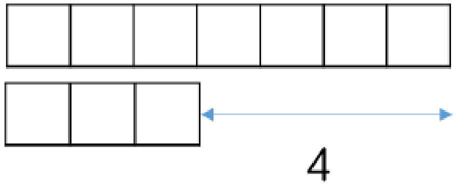
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

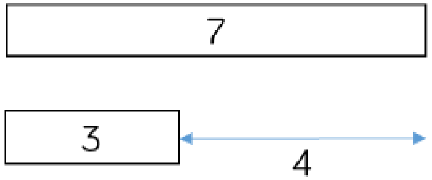


$7 + 3 = 10$

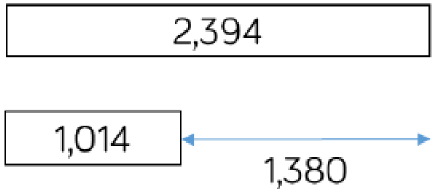


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

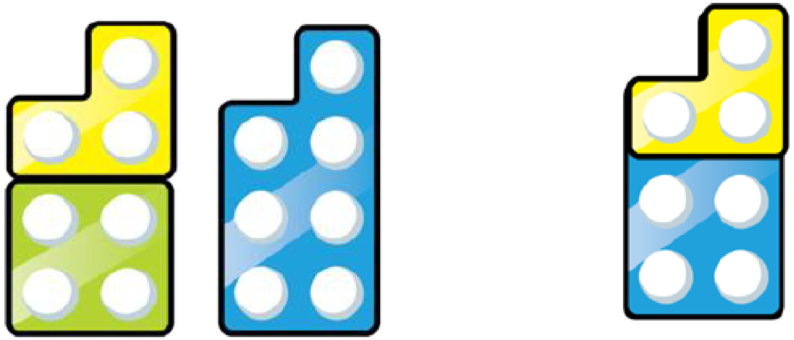
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

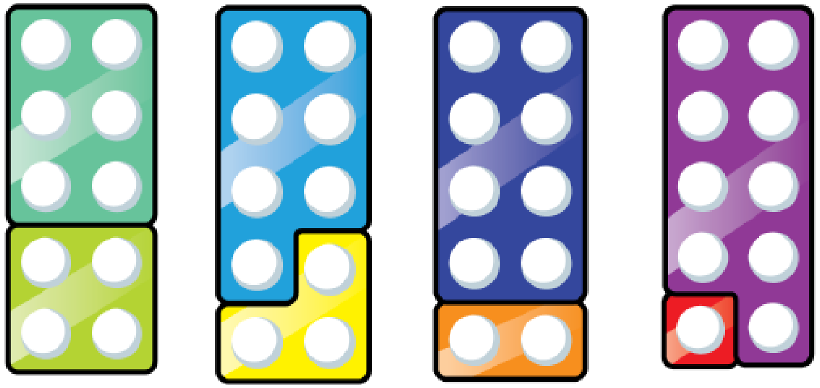
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Numicon



7 = 4 + 3   7 = 3 + 4   7 − 3 = 4



6+4   7+3   8+2   9+1

Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

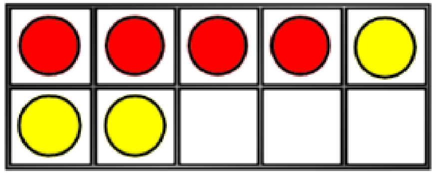
When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

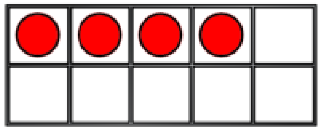
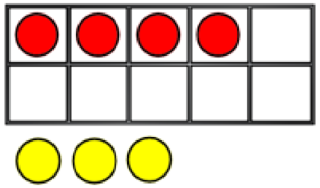
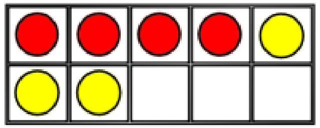
Ten Frames (within 10)

Five frame for FS1 and 2

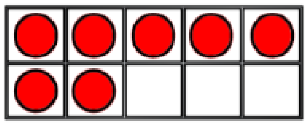
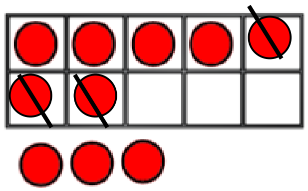
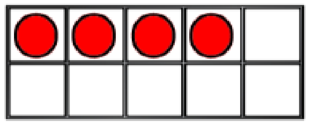


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.

First	Then	Now
		

$4 + 3 = 7$

First	Then	Now
		

$7 - 3 = 4$

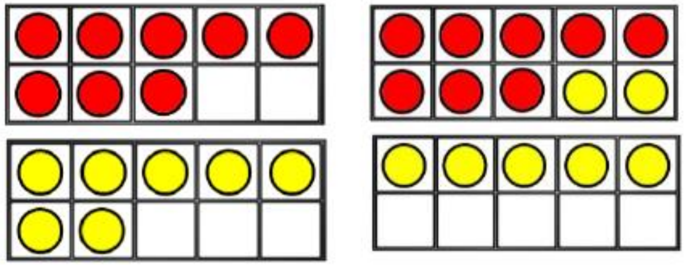
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.
Using these structures, the ten frame can enable children to find all the number bonds for a number.

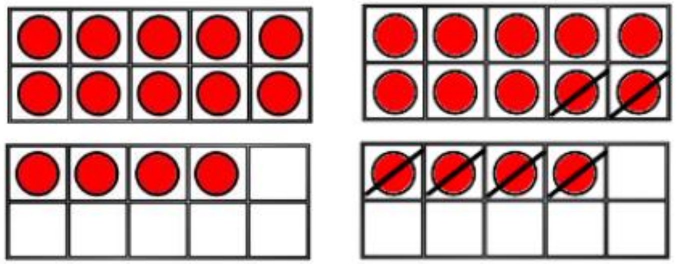
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



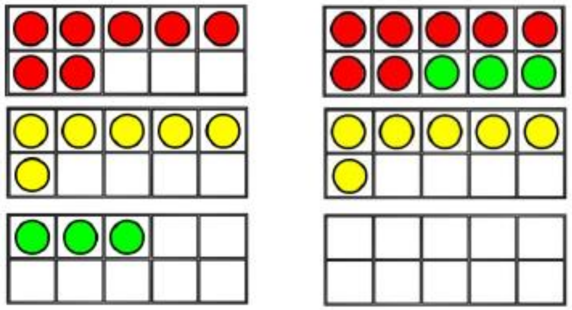
$$8 + 7 = 15$$

2 5



$$14 - 6 = 8$$

4 2



$$7 + 6 + 3 = 16$$

10

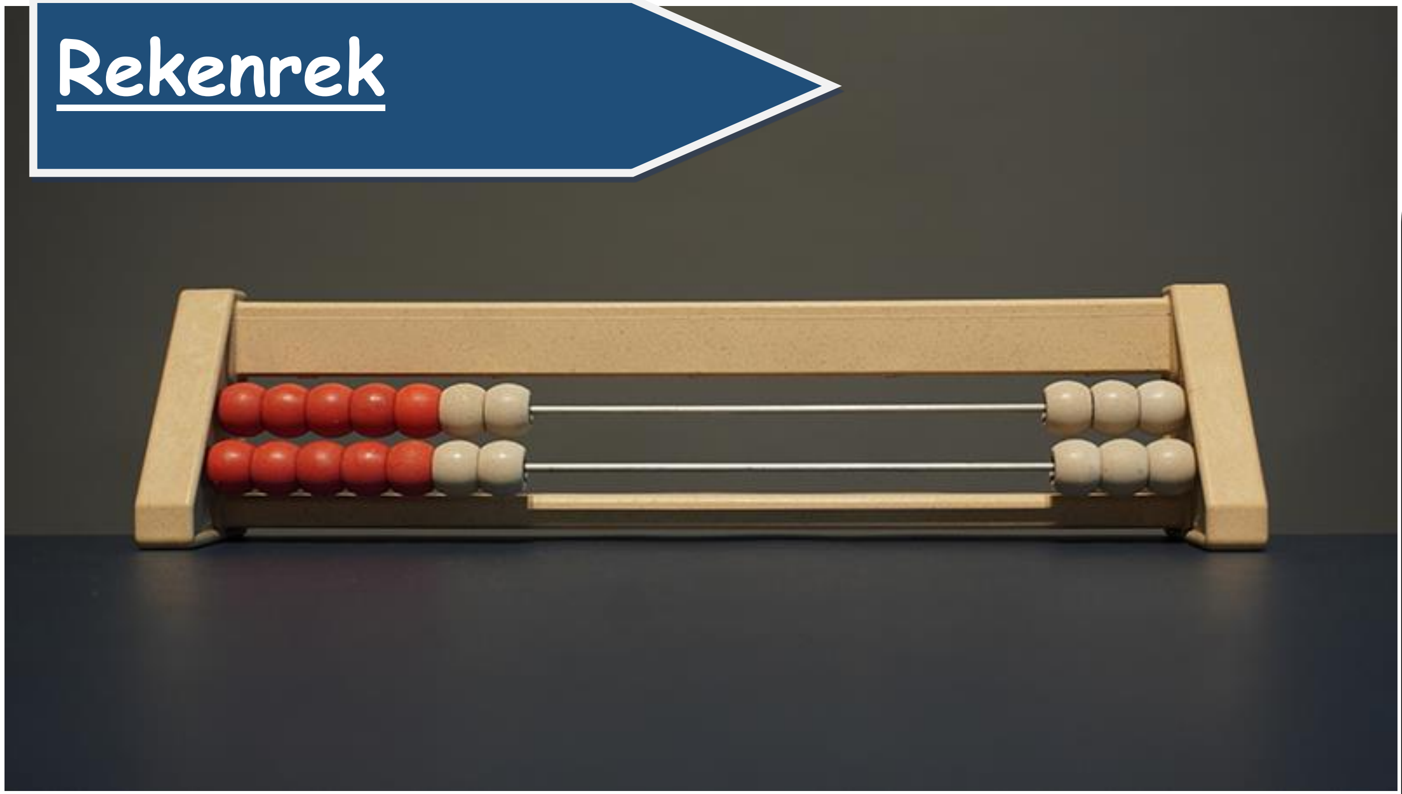
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Rekenrek



Benefits

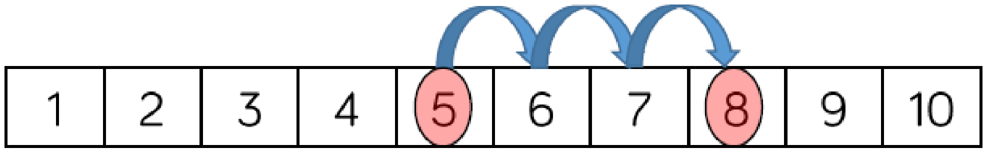
The red and white beads build upon children's ability to subitise to 5 due to the arrangement of the red and white beads. 7 is a 5 and a 2 Children start to benchmark to 5 and 10.

Beads start in the 'ready position' towards the right. Children are taught to push groups of beads all at once rather than one by one to develop more efficient addition and subtraction strategies.

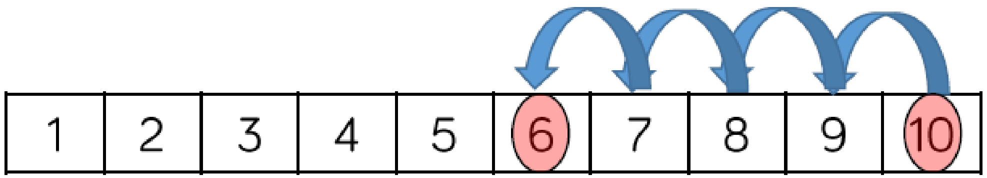
They are especially useful for developing an understanding of teens numbers as 'ten and some ones' and then also for calculation strategies involving bridging through ten.

Number Tracks

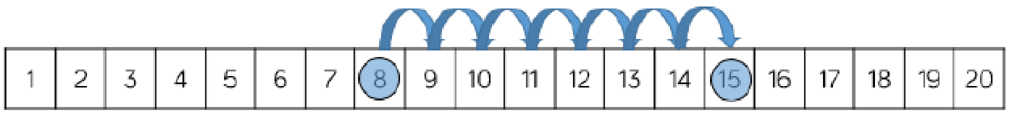
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

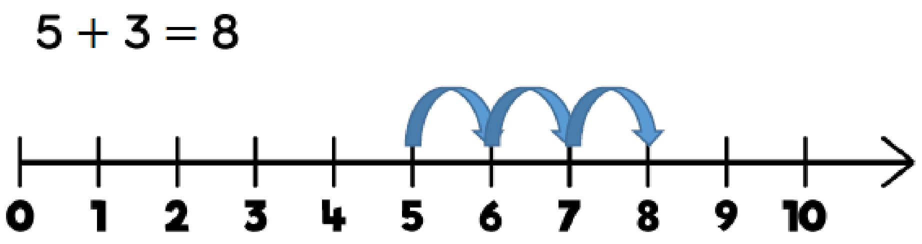
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

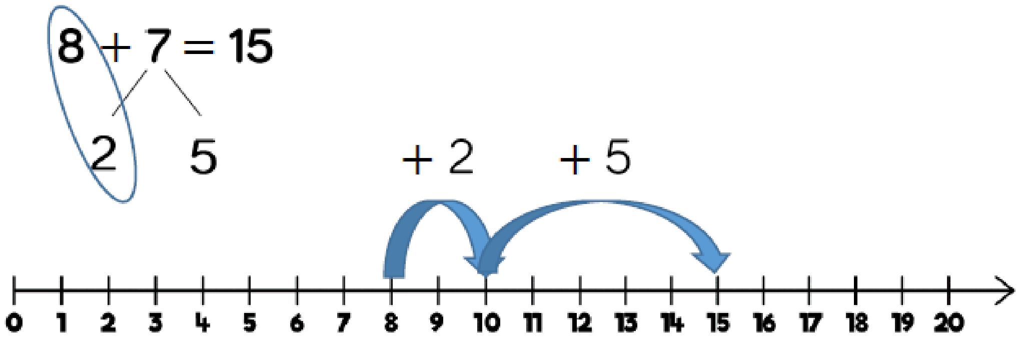
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

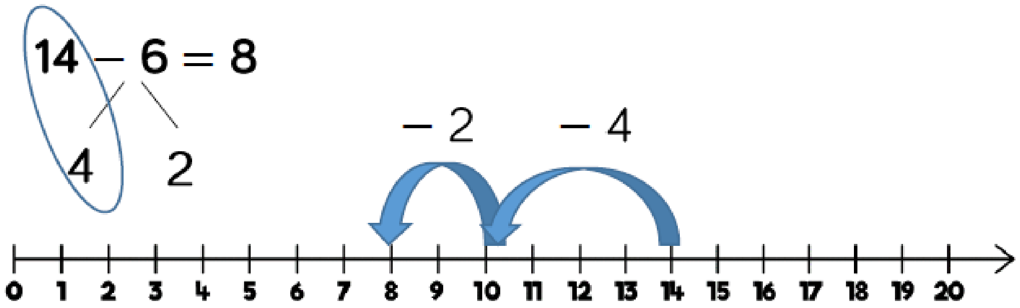
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)







Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

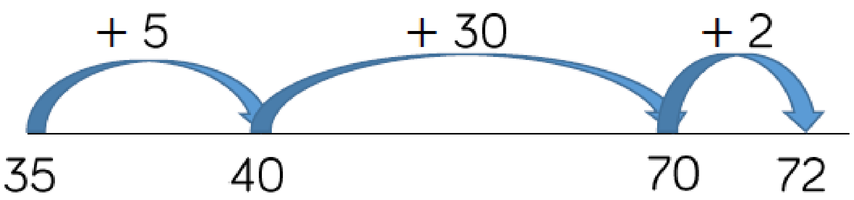
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

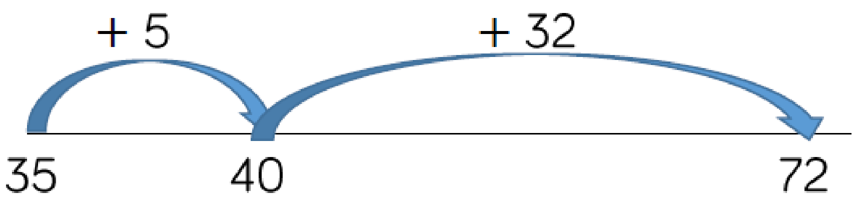
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

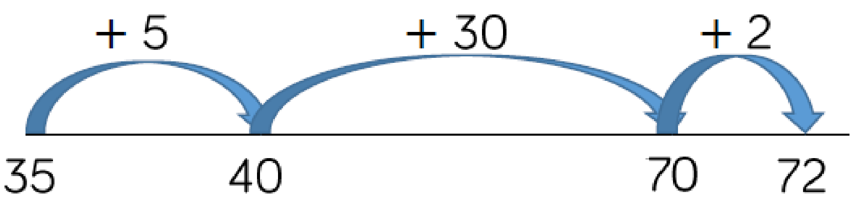
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



Benefits

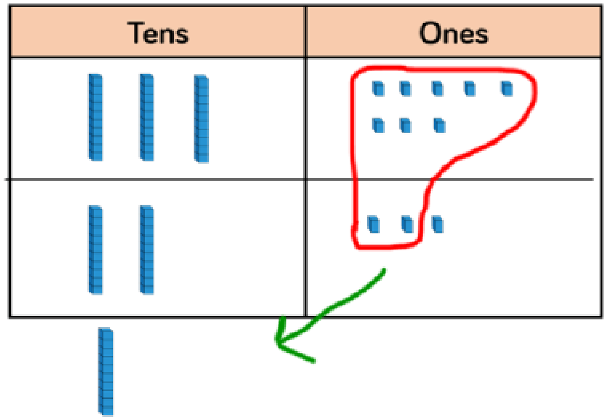
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

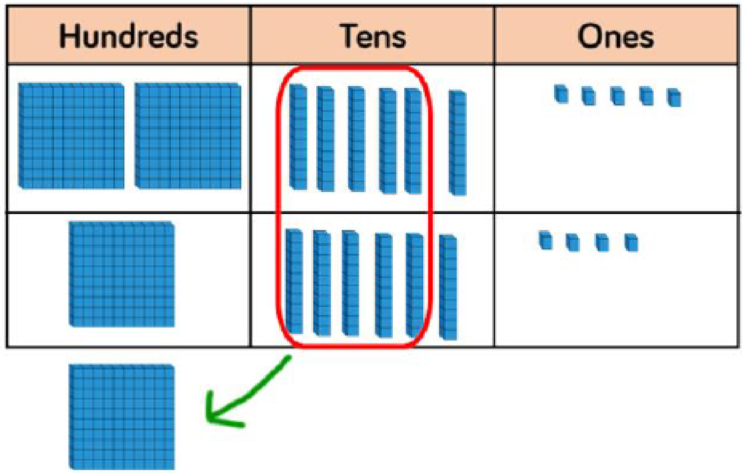
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline {\scriptstyle 1\ \ } \end{array}$$

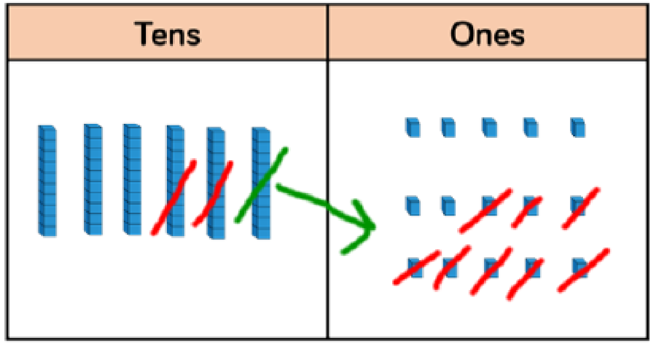
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

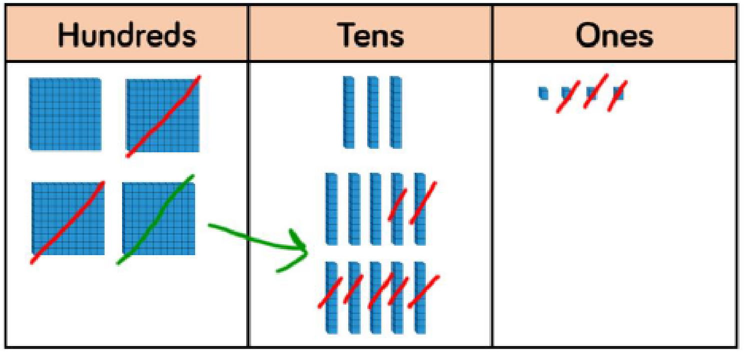
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$



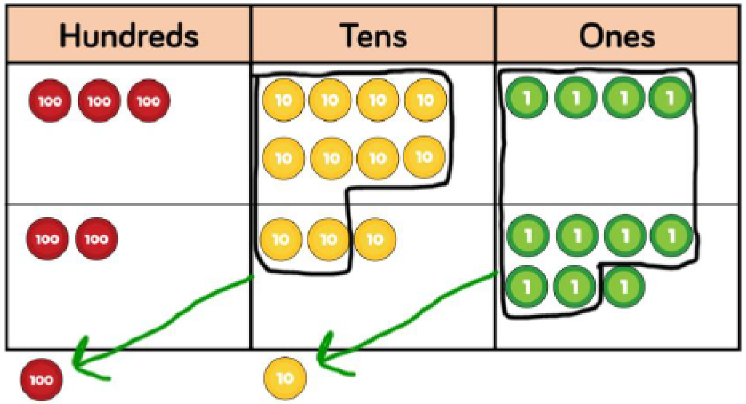
$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}35 \\ -\ 273 \\ \hline 262 \\ \hline \end{array}$$

Benefits

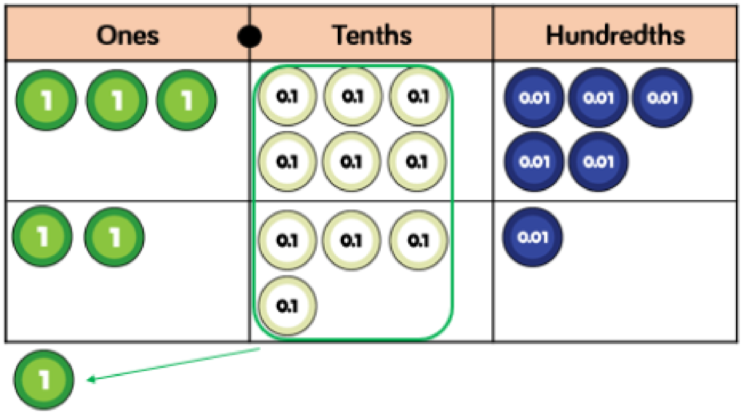
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ 1\ \ \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1\ \ \ \ \ \end{array}$$

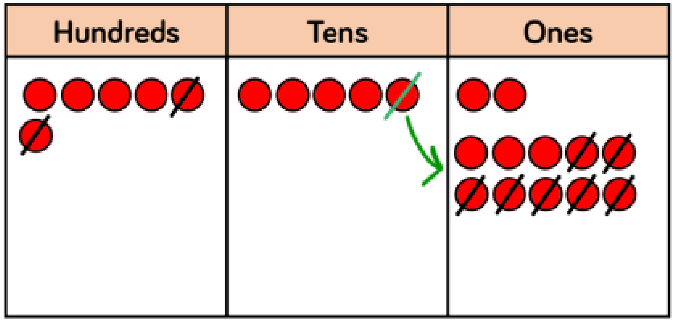
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

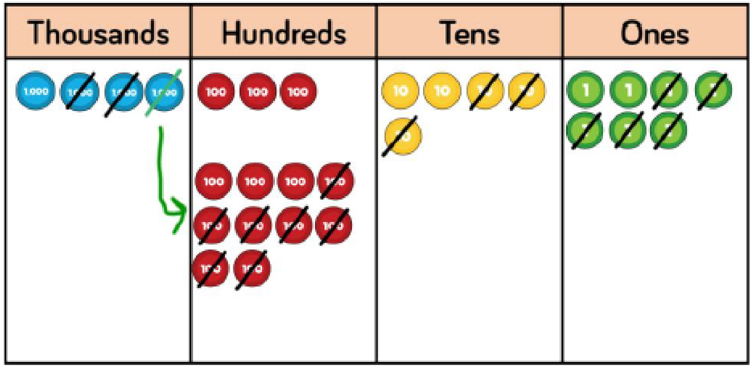
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} 6\overset{4}{\cancel{5}}{}^{1}2 \\ -\ 207 \\ \hline 445 \\ \hline \end{array}$$



$$\begin{array}{r} \overset{3}{\cancel{4}}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \\ \hline \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

EYFS Framework and Development Matters 2021

"Children should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers. By providing frequent and varied opportunities to build and apply this understanding - such as using manipulatives, including small pebbles and tens frames for organising counting - children will develop a secure base of knowledge and vocabulary from which mastery of mathematics is built."

FS1

Objectives Age 3-4	Activities	Common Misconceptions/errors
• Recite numbers past 5.	• Say number words in sequence • (Extension - starting at different numbers; and counting backwards) • Nursery rhymes and actions	Confusion with 'teen' numbers and 'ty'
• Say one number for each item in order: 1,2,3,4,5. • Know that the last number reached when counting a small set of objects tells you how many there are in total ('cardinal principle'). • Show 'finger numbers' up to 5.	• **Tagging each object with one number word (relating counting sequence to cardinality)** • **Wide range of opportunities to develop 1:1 correspondence** • Count things in a straight line • Count things of different sizes – this helps children to focus on the numerosity of the count • Count things that can't be seen, such as sounds, actions, words • Count things that cannot be moved, such as pictures on a screen, birds at the bird table, faces on a shape. • Count things in irregular arrangements • Conservation of number – correct a [puppet who says there are less objects because rearranged • Explore cardinality of 5 – link to dice patterns and fingers • Rolling a dice and collecting that many objects • Counting along a track e.g. games	• Missing out a number or saying twice • Grabbing any number or failing to stop at number asked for • Re-counting when a set number of objects is rearranged
• Link numerals and amounts: for example, showing the right number of objects to match the numeral, up to 5. • Experiment with their own symbols and marks as well as numerals.	• Matching Numicon, objects and five frames to numerals • Tidying away e.g. 4 pencils in pots, 3 trucks on shelf • Matching numerals to quantities they can subitise	

• Develop fast recognition of up to 3 objects, without having to count them individually (‘subitising’).	• Identifying without counting, dots or objects up to 3 • Play hidden object games where objects are revealed for a few seconds e.g. toys under a bowl • “All at once” fingers – show 4 fingers • Numberblock recognition	
• Compare quantities using language: ‘more than’, ‘fewer than’	• Collections to sort and compare • Collections with the same amount • Making two collections have the same amount	
	• Children can also apply this to ‘composition’ tasks. Learning to see numbers as a whole number and its parts is a key development in number understanding and partitioning and putting back together underpins addition and subtraction. • See other numbers in Number blocks • Make a Numicon number from other Numicon pieces • Investigate ways to make a rod of 5 cubes. • Stand 5 skittles and knock some over. How many up and how many down? • Splitting a number of objects between two plates	

FS2

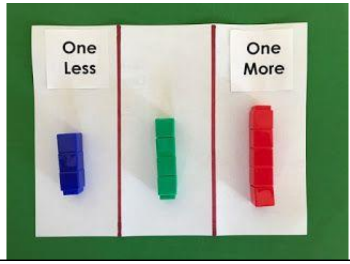
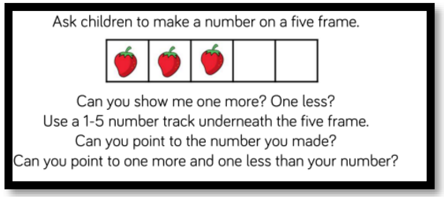
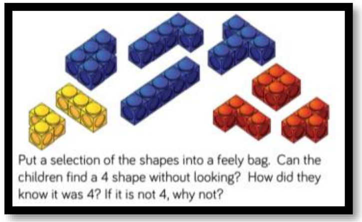
Prior to calculations, the following concepts are important, as described in Developmental Matters 2021 and EYFS Framework 2021. The Mastery in Numbers program will help develop these through 4 main areas:

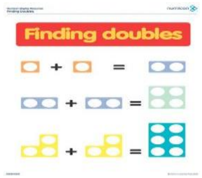
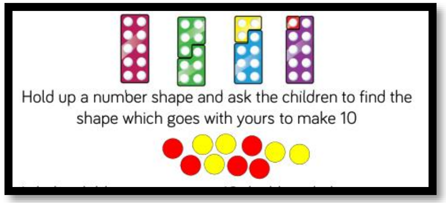
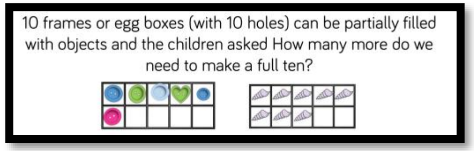
subitising, composition of number, cardinality and ordinality, and comparison.

Key representations used are the Rekenrek and the tens frame

Objectives Age 4-5	Activities	Common Misconceptions/errors
• Count objects, actions and sounds • Counting beyond 10	• continue to develop verbal counting to 20 and beyond • continue to link counting to cardinality, including using their fingers to represent quantities between 5 and 10 • become more familiar with the counting pattern beyond 20. continue to develop verbal counting to 20 and beyond, including counting from different starting numbers • continue to develop confidence and accuracy in both verbal and object counting.	
• Subitise	• Show small quantities in familiar patterns (for example, dice) and random arrangements. • Play games which involve quickly revealing and hiding numbers of objects. • Put objects into five frames and then ten frames to begin to familiarise children with the tens structure of the number system. Benchmark numbers to 5 and 10 “I see a row of 5 and 1 more so that equals 6” “I see 1 less than 10 so that must be 9” • Prompt children to subitise first when enumerating groups of up to 4 or 5 objects: “I don’t think we need to count those. They are in a square shape so there must be 4.” Count to check. • Encourage children to show a number of fingers ‘all at once’, without counting. • explore symmetrical patterns, in which each side is a familiar pattern,linking this to ‘doubles’.  	•
• Link numeral with cardinal value	• order numbers, linking cardinal and ordinal representations of number	

• Compare numbers	• compare sets using a variety of strategies, including 'just by looking', by subitising and by matching • compare sets by matching, seeing that when every object in a set can be matched to one in the other set, they contain the same number and are equal amounts. • explore ways of making unequal sets equal. • order sets of objects, linking this to their understanding of the ordinal number system.	children not comparing the numerosity of the group and considering more in terms of size • children giving a response that does not match the context when estimating a number; e.g. when adding, giving as an answer a number that is smaller than the numbers given. Example: 'There are 7 cars in a garage and then 2 more go in.' The child guesses there are 4 cars in total inside
• One more/less	• labelling groups with the correct numeral. Do children spot the error if a group is mislabelled? For example, 'The label on the pot says 4 and we have 5 – what do we need to do?' A child may say, 'We need to take one out because we have one too many.' • ensuring children focus on the numerosity of the group by having items in the collection of different kinds and sizes • making predictions about what the outcome will be in stories, rhymes and songs if one is added to, or if one is taken away • Support children in recognising that if they add one, they will get the next number, or if one is taken away, they will have the previous number. **Language:**	

	'My number is………one more than my number is……' **Language:** 'My number is……one less than my number is……'   	
• Composition of numbers to 10	• Focus on composition of 2, 3, 4 and 5 before moving onto larger numbers - practise recalling 'hidden parts' • begin to see that numbers within 10 can be composed of '5 and a bit'  • Provide a range of visual models of numbers: for example, six as double three on dice, or the fingers on one hand and one more, or as four and two with ten frame images. • Model conceptual subitising: "Well, there are three here and three here, so there must be six." • Emphasise the parts within the whole: "There were 8 eggs in the incubator. Two have hatched and 6 have not yet hatched." • Plan games which involve partitioning and recombining sets. For example, throw 5 beanbags, aiming for a hoop. How many go in and how many don't? • Begin to explore the composition of odd and even numbers. •	

• Automatically recall number bonds for numbers 0-5 and some to 10 • Doubles to 5+5	• Have a sustained focus on each number to and within 5. Make visual and practical displays in the classroom showing the different ways of making numbers to 5 so that children can refer to these. • Help children to learn number bonds through lots of hands-on experiences of partitioning and combining numbers in different contexts, and seeing subitising patterns.  • Play hiding games with a number of objects in a box, under a cloth, in a tent, in a cave, etc.: “6 went in the tent and 3 came out. I wonder how many are still in there?” • Intentionally give children the wrong number of things. For example: ask each child to plant 4 seeds then give them 1, 2 or 3. “I’ve only got 1 seed, I need 3 more.” • Spot and use opportunities for children to apply number bonds: “There are 5 of us but only 2 clipboards. How many more do we need?” • Place objects into a five frame and talk about how many spaces are filled and unfilled. • Understand the effect of adding or subtracting zero	

Addition

Mental methods of calculation are practised on a regular basis and secured alongside the learning and use of written methods of addition. Informal methods such as partitioning and other jottings will be used to develop mental methods and support the place value understanding behind the traditional column approach.

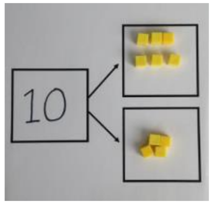
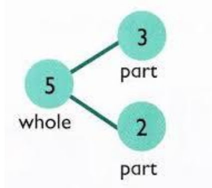
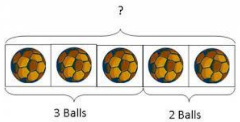
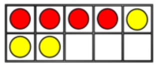
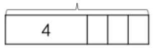
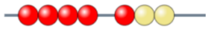
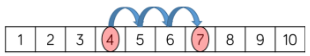
Children use mental methods when appropriate, but for calculations that they cannot do in their heads efficiently, they use a formal written method accurately and with confidence, initially supported by models such as Numicon, Ten Frames, Rekenrek, Place Value Counters and Base Ten blocks (Dienes).

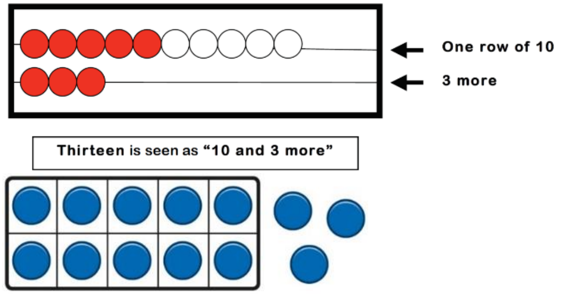
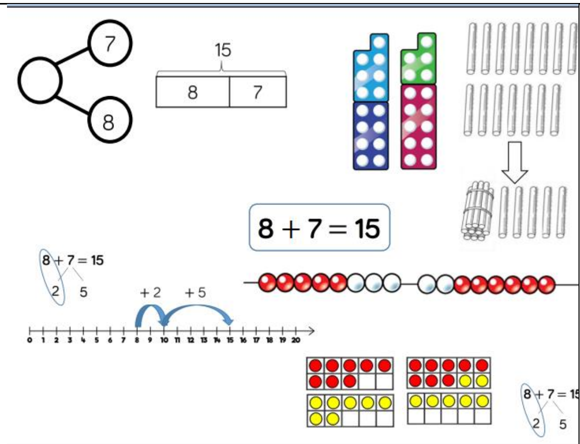
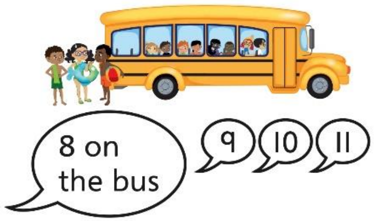
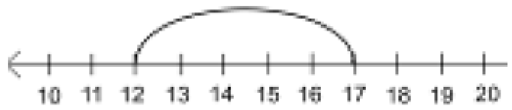
There are some key basic skills that children need to help with addition, which include:

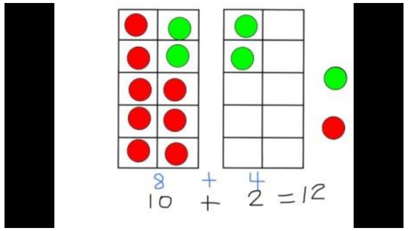
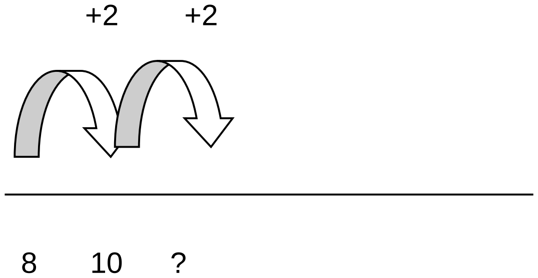
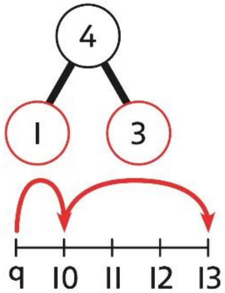
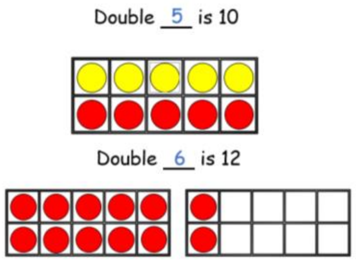
- Subitising (knowing a number without counting)
- counting
- doubles and nearly doubles (if I know 4 + 4 = 8 then I know 4 + 5 = 8 + 1 more)
- knowing number facts **within** 10
- recalling all number bonds **to** 10
- recalling number bonds to 20 and 100 (7 + 3 = 10, 17 + 3 = 20, 70 + 30 = 100)
- adding mentally a series of one-digit numbers (5 + 8 + 2) (look for bonds to 10, doubles)
- adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12
- understanding and using addition and subtraction as inverse operations

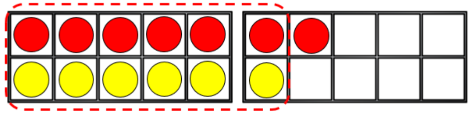
<u>Year 1 Addition</u>

<u>Running alongside this progression is our weekly work on the Mastering Numbers Project which focuses on the underpinning of the maths skills and concepts needed for addition and subtraction: subitising, composition of number, cardinality and ordinality, and comparison, using the Rekenrek.</u>

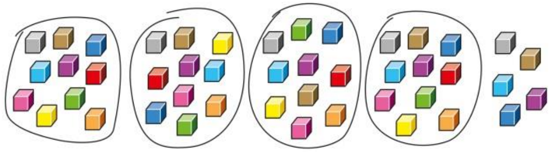
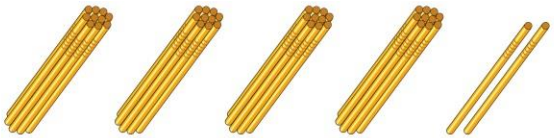
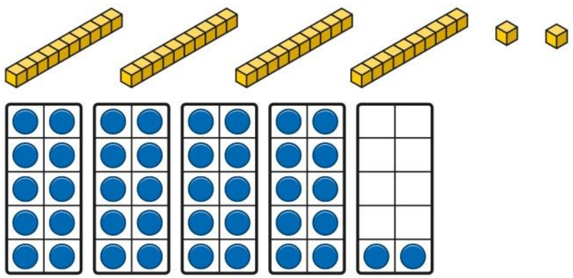
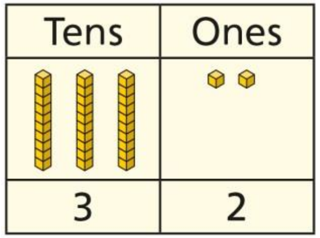
Objective and Strategies	Concrete	Pictorial	Abstract
Adding two 1-digit numbers Combining two parts to make a whole: part-whole model (aggregation) And growing things/increasing by an amount (augmentation)	  Use cubes to add two numbers together as a group or in a bar.  	  Use pictures to add two numbers together as a group or in a bar.     	4 + 3 = 7 10= 6 + 4 Be sure to use equals sign in various places Remember to review effects of adding zero.

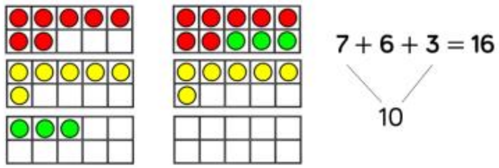
Understand teen numbers as a complete ten and some more **Adding a two digit number and a 1 digit number**	Rekenrek https://apps.mathlearningcenter.org/number-rack/ (online version)  	 	*1 ten and 3 ones equal 13.* *10 + 3 = 13*
Starting at the bigger number and **counting on**	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 Numberlines 12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 17 =12 + □ Place the larger number in your head and count on the smaller number to find your answer.

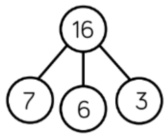
Bridging through 10. (for numbers to 20) **(pre-requisite skills: partition one digit numbers and know number bonds to ten)**	Also use Rekenrek to bridge through the ten 8 + 4 = Ten frames Start with the bigger number and use the smaller number to make 10. 	  Can link to part whole model to support	8+ 4 = 8 + 2 + 2 = 12 If I am at eight, how many more do I need to make 10? How many more do I add on now?
Doubles and near doubles	 Encouraging use of ten frame to see 5 + 5 =10 then another 2 more	This could be represented with pictures of ten frames	6 + 6 = 12 ? = 6 + 6 6 + 7 = 12 ? = 6 + 7

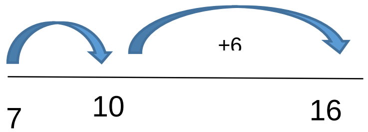
	Complete the sentences.  7 + 6 is equal to double _6_ plus 1 Double _6_ = _12_ 7 + 6 = _13_		

Year 2 Addition

Objective and Strategies	Concrete	Pictorial	Abstract
See Year 1 Bridging through 10. (for numbers to 20)			
Understand tens and ones	Group objects into 10s and 1s.  Bundle straws to understand unitising of 10s.  Understand 10sand 1s equipment 	Represent numbers on a place value grid, using sketches and numerals.  	Tens | Ones 4 | 3

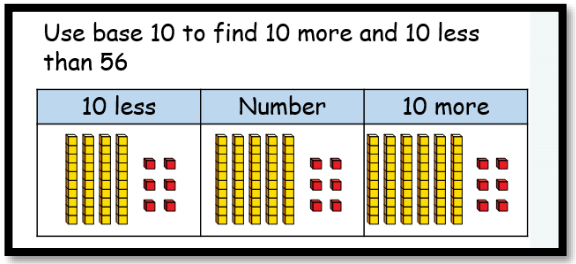
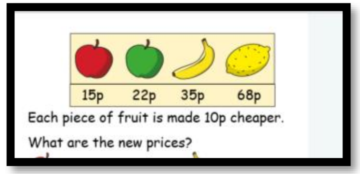
<table>
<tr>
<td>Adding three single digits</td>
<td>

Also encourage looking for doubles or near doubles</td>
<td>

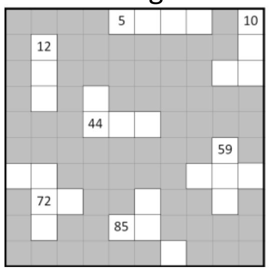
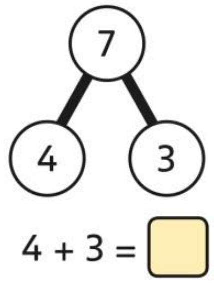
Numberline


Bar model
</td>
<td>$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$
$= \boxed{17}$

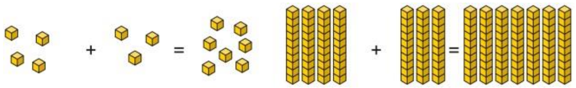
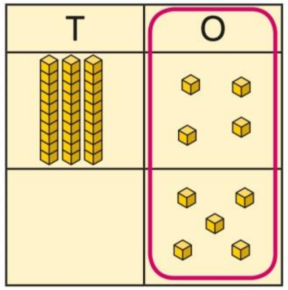
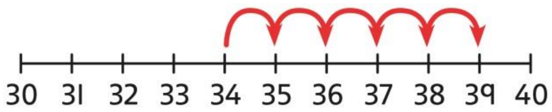
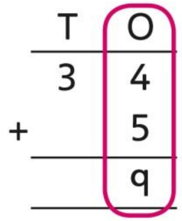
Reinforces commutativity in addition

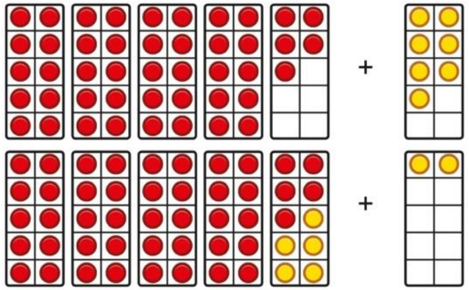
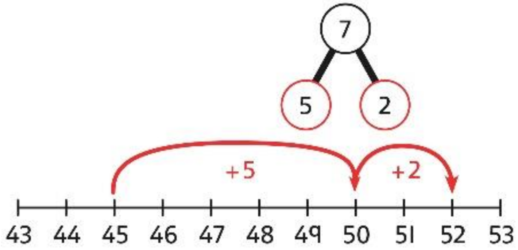
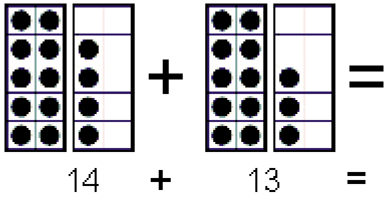
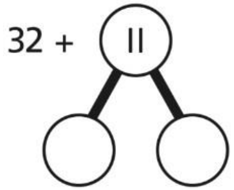
Combine the two numbers that make 10 and then add on the other number.</td>
</tr>
<tr>
<td>10 more/less</td>
<td>
</td>
<td>Drawing Base 10
Locating on number grid</td>
<td>27+ 20 =
42 = ___ + 10

</td>
</tr>
<tr>
<td>Adding multiples of 10</td>
<td>Use known bonds and unitising to add 10s.

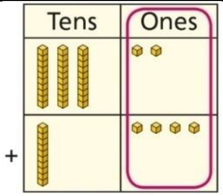
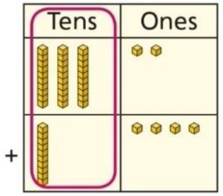

I know that 4 + 3 = 7.
So, I know that 4 tens add 3 tens is 7 tens.</td>
<td>Sketching Dienes

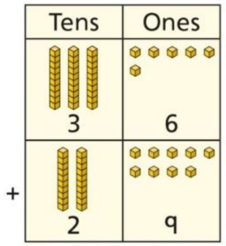
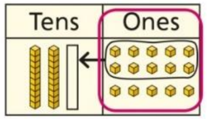
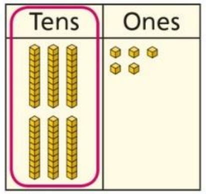
Use of the hundred square to consolidate mental image i.e. counting down adds a ten

</td>
<td>Use known bonds and unitising to add 10s.


4 + 3 = 7 so 4 tens + 3 tens = 7 tens</td>
</tr>
</table>

			
Adding a 1-digit number to a 2-digit number not bridging a 10	Add the 1s to find the total. Use known bonds within 10.  *41 is 4 tens and 1 one.* *41 add 6 ones is 4 tens and 7 ones.* This can also be done in a place value grid. 	Sketching of Dienes Use of a numberline Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy. 	This can be represented horizontally or vertically. *34 + 5 = 39* *or*  Progressing to: Missing numbers and equivalence 35 = $\boxed{}$ + 5 14 + 5 = 10+$\boxed{}$

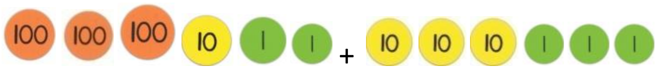
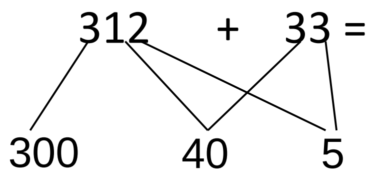
Adding a 1-digit number to a 2-digit number bridging 10	Apply 'making a ten' strategy 45 + 7 = 45 + 5 + 2 		*45 + 7 =* *45 + 5 + 2 = 52*
Adding two 2- digit numbers by partitioning to support mental methods (no exchange)	First without exchange. Use Dienes or Ten Frames  Add the tens and ones separately. Either: 10 + 10 + 4 + 3 Or 14 + 10 + 3	Add the 10s and 1s separately. Use a part-whole model to support.  *11 = 10 + 1* *32 + 10 = 42* *42 + 1 = 43* *(or add the tens and then the ones)* Can also use a numberline. Builds upon the adding ten strategy and making ten strategy	32 + 11 = 32 + 10 + 1 Or 30 + 10 = 40 2 + 1 = 3 40+ 3 =43

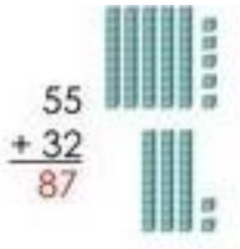
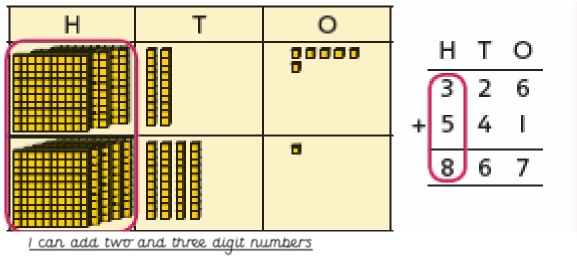
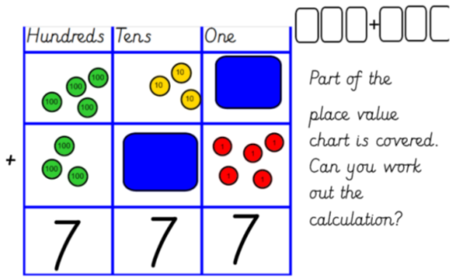
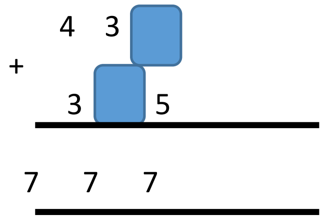
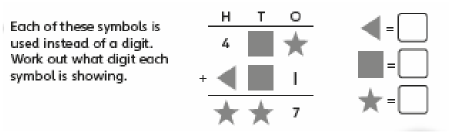
Adding two 2 digit column method (no exchange) This is taught as an efficient method for adding larger numbers (even though WR have moved away from it)	  Add the ones first then the tens Important to do this **alongside the written** method so children see the link.	Represent this with sketches of dienes	T O 3 2 + 1 4 6 T O 3 2 + 1 4 4 6 Reasoning Progressing to: Missing numbers 4 2 + $\boxed{}$ 6 3
Adding two 2- digit column method with exchange (with GD)	Add the 1s. 10 ones for a ten. Then add the 10s. Use Dienes as opposed to counters. Formal recording of this is GD	Can draw dienes and circle 10 to be exchanged	Leads to formal representation NOTE: Use place value knowledge to support mental method jottings as an alternative to support mental calculations 48 + 36 40 + 30 = 70 8 + 6 = 14

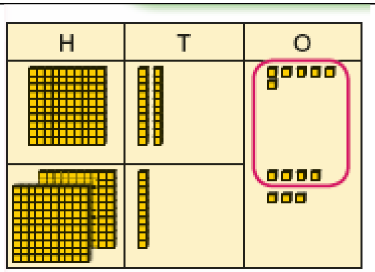
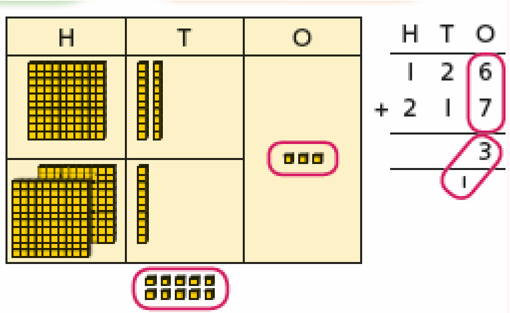
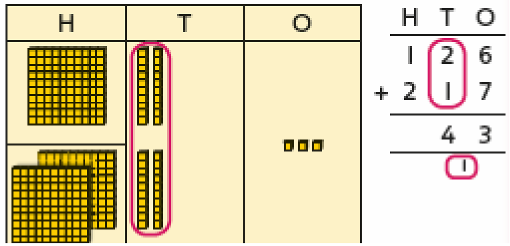
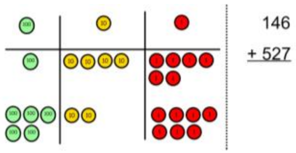
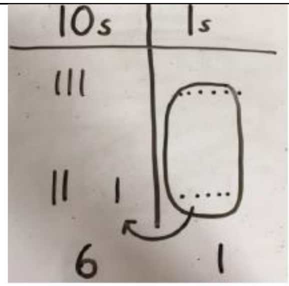
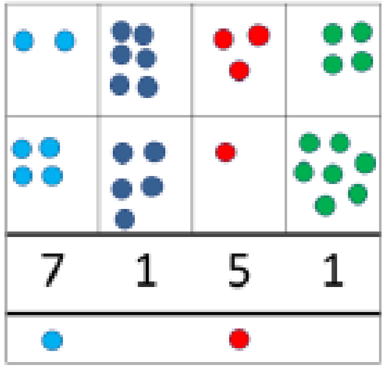
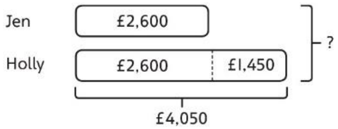
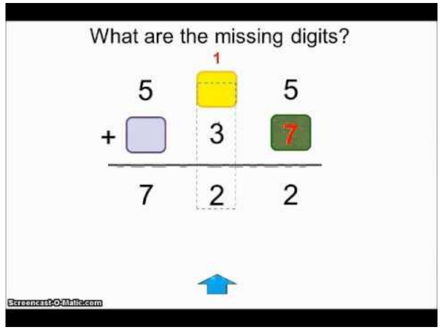
			36 + 29 65

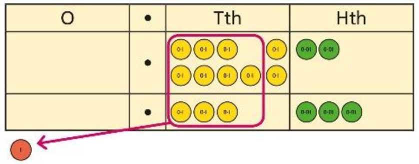
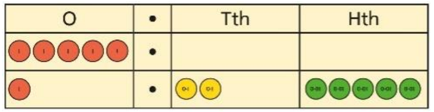
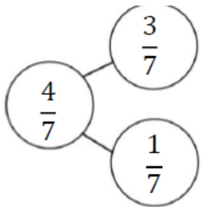
Years 3 - 6 Addition

(with increasingly larger numbers)

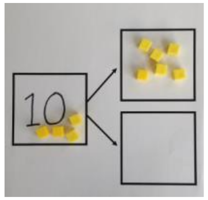
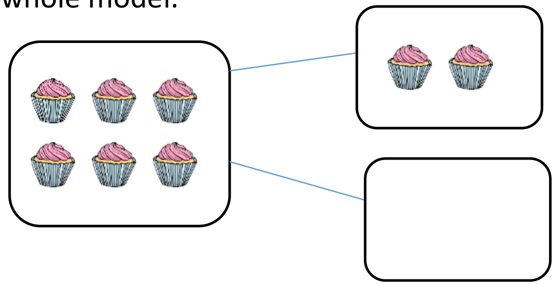
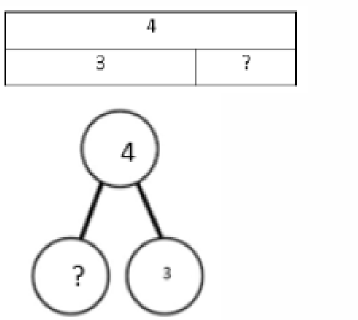
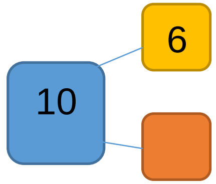
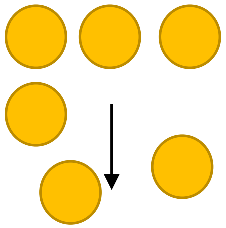
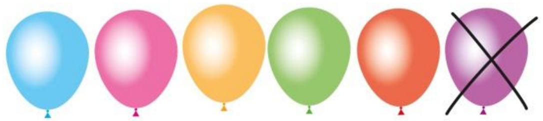
Objective and Strategies	Concrete	Pictorial	Abstract
Adding 3 digit plus ones then 3 digit plus two digit, then 3 digit plus3 digit (to support **mental** methods)	 312 + 33 =	Draw counters Bar model to show parts make a whole Use mental jottings to help e.g. numberlines, kite strings with partitioning and bar models 	Use number bonds to add the 1s and understand that this is more efficient and less prone to error than counting on in ones. *245 + 4 = ?* *I will add the 1s.* *5 + 4 = 9* *So, 245 + 4 = 249* Include missing numbers 320 + $\square$ = 450 (recognising that this can be done mentally by using kowledge of place value and counting up) 320 + 80 + 50 80+50 = 130 Or I need 1 more hundred and 3 more tens 140 + 5 = 100 + $\square$ Understand commutative property

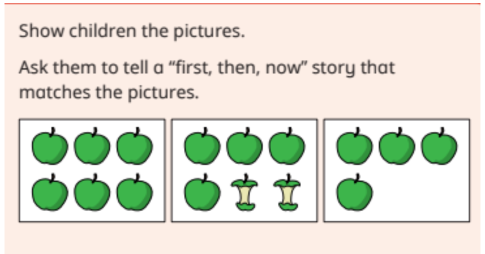
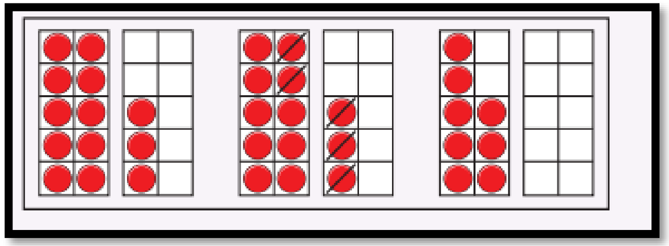
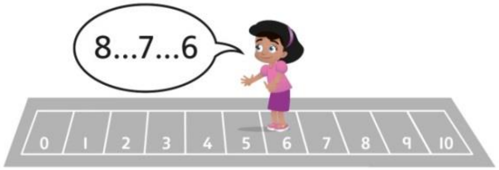
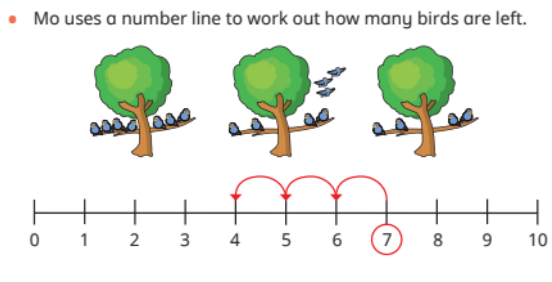
Column method- no exchange Addition of 3 addends too	Add together the ones first then add the tens. **Use the Base 10 blocks first before moving onto place value counters.**   	After practically using the base 10 blocks and place value counters, children can draw the counters or Base Tens to help them to solve addition.	Formal written method of three digit colum addition, lining up digits carefully. **Ensure additon of numbers with a different number of digits is included.** **Also ensure zero is used.** 344 + 55 Progressing to: Missing digits  

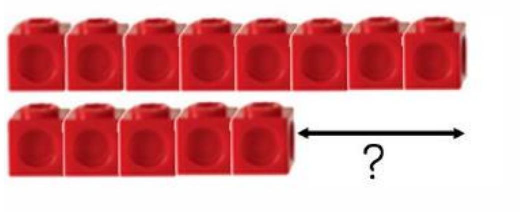
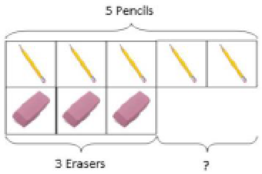
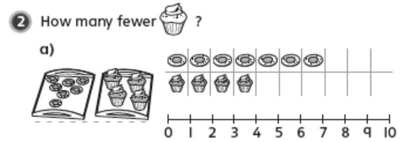
Objective and Strategies	Concrete	Pictorial	Abstract
Column method- with exchange Column addition remains efficient when used with larger whole numbers or decimals, and when adding more than two numbers. Once learned, the method is quick and reliable. Include addition of <u>3 addends</u>	    Add up the ones and exchange 10 ones for one 10. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.	 Children can draw a pictoral representation of the columns and B10 or place value counters to further support their learning and understanding.  Bar models may be used as a visual representation 	. $\begin{array}{r} 148 \\ +\ \ 36 \\ \hline 184 \\ \hline {\scriptstyle 1} \end{array}$ **NOTICE THE PLACEMENT OF THE EXCHANGED 10 – UNDER THE LINE**. 

Y5 Addition with decimals	 O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 1 As children move on to decimals, money and decimal place value counters can be used to support learning. Ensure children understand about making a whole. Include examples where the numbers of decimal places are different. 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2  O · Tth Hth 5 · 0 0 + 1 · 2 5 6 · 2 5		*3.4 + 0.65 = ?* O · Tth Hth 3 · 4 0 + 0 · 6 5 ·
Use of Part whole with FDP		 	

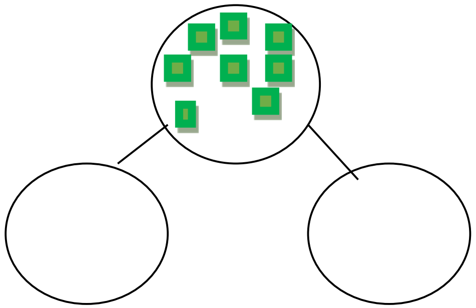
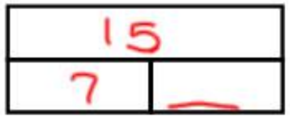
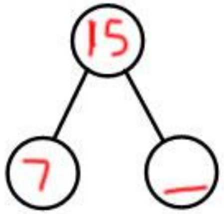
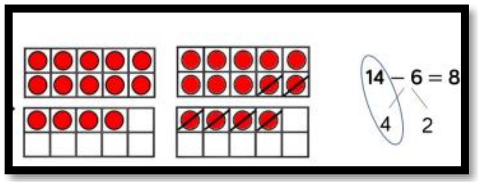
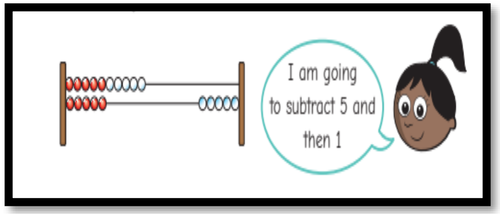
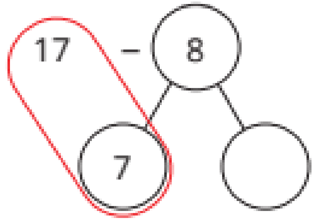
Subtraction in Year 1

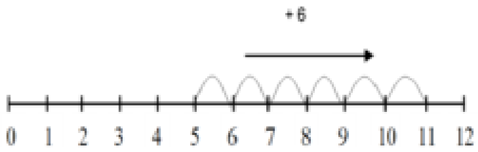
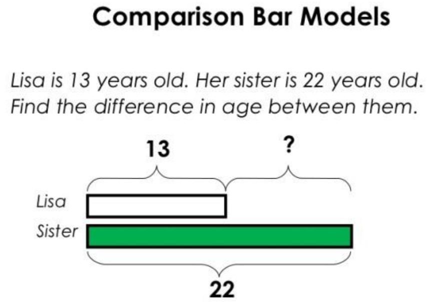
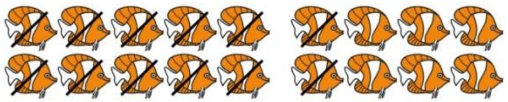
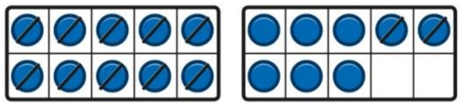
Objective and Strategies	Concrete	Pictorial	Abstract
Part Part Whole Model **Finding a missing part, given a whole and a part**	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? Start with the ten in the whole box then physically move a part into the part box, then find the other part. 10-6 =	Use a pictorial representation of objects to show the part part whole model.  	 Move to using numbers within the part whole model. Relate + - 6 + 4 = 10 10 = 4 + 6 10 – 6 = 4 10 – 4 = 6
Taking away ones (remember to include – 0. Start with numbers within 10 then later within 20)	Use physical objects, counters, cubes etc to show how objects can be taken away by physically moving them.  6 – 2 = 4	Move to crossing out drawn objects to show what has been taken away. Vary the language: *1 less than 6 is5. 6 subtract 1 is 5*. 	6 – 2 = 4

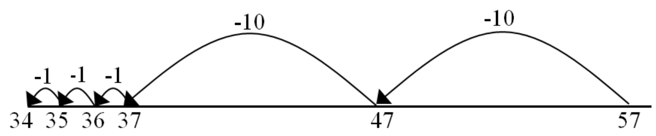
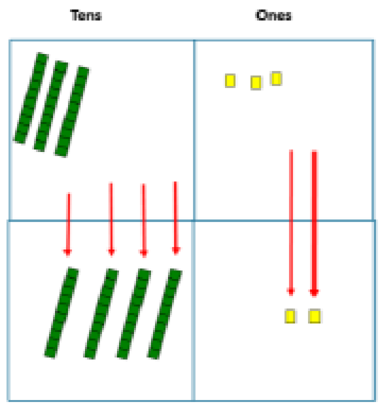
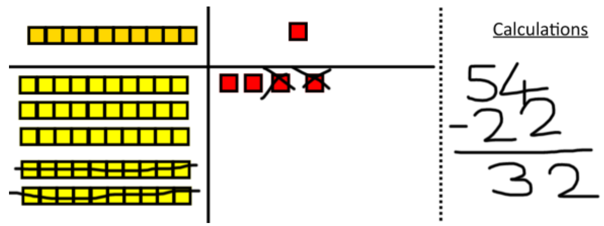
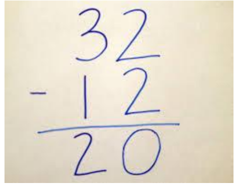
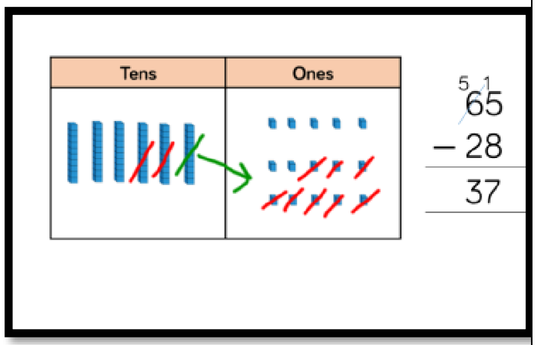
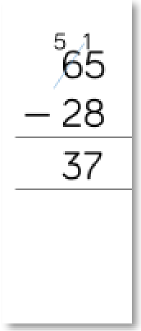
		 Use first, then , now language 	
Counting back	Use counters and move them away from the group as you take them away counting backwards as you go.  	Count back on a number line or number track. Make sure children **count jumps.** 	7 – 3 =
Find the **difference** (up to 20)	Compare amounts and objects to find <u>the difference</u>.	Count on to find the difference.	Hannah has 20 sandwiches, Helen has 15 sandwiches. How

(counting up is often easier) NOTE: using the terminology ‘difference’ is important. This is a word that can easily be misunderstood.	  Use cubes to build towers or make bars to find the difference	Use basic bar models with pictures to find the difference. Can then move to continuous bar model   	many more does Hannah have? Use words **more and fewer** 

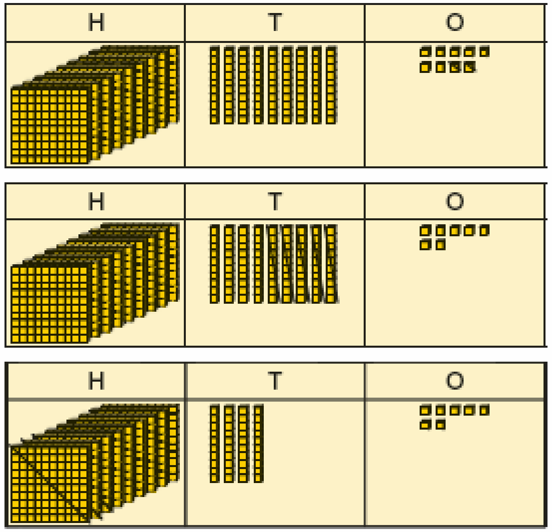
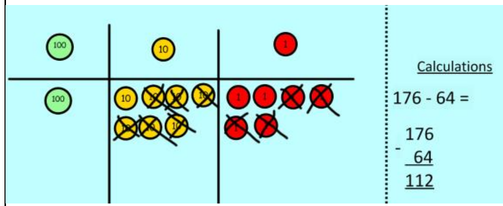
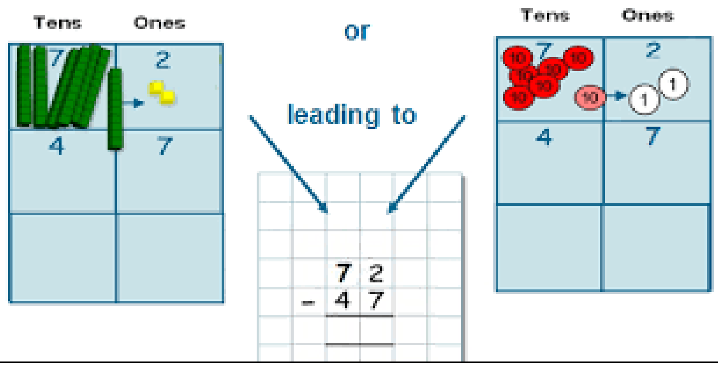
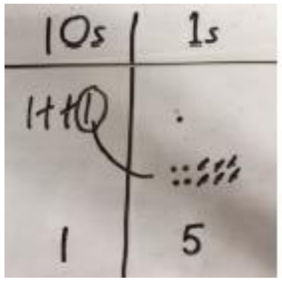
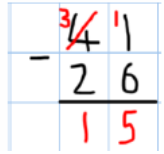
Subtraction in Year 2

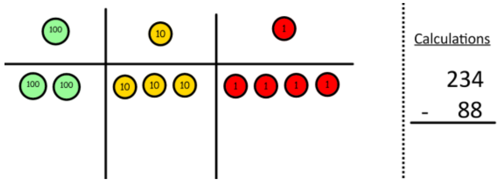
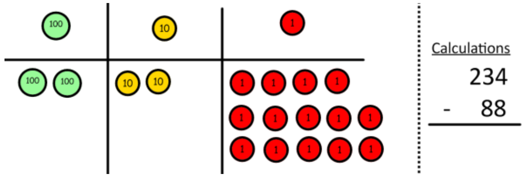
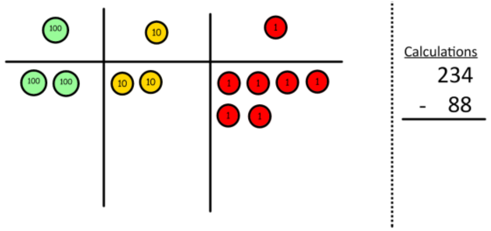
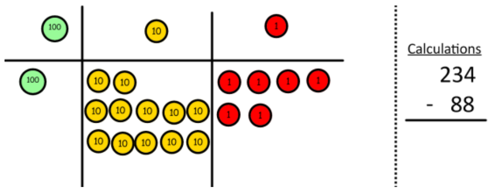
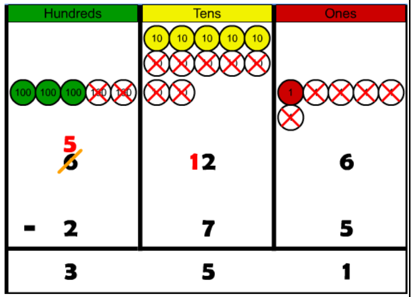
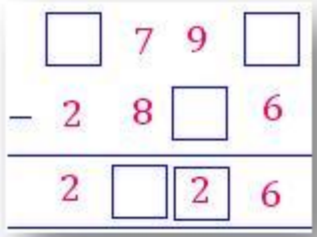
As with Y1 Part Part Whole Model (as above but with increasingly larger numbers)		Linking with addition and with known facts   7 – 5 = 2 So 7tens – 5 tens = 70 - 50 = ?	Missing numbers 25 - $\square$ = 17
Subtract across a ten (also used with larger numbers)	 15-6  	May draw ten frame and cross out or may draw numberline and partition through the ten. Partition subtrahend using cherry model  	17-8= 17-7-1 =

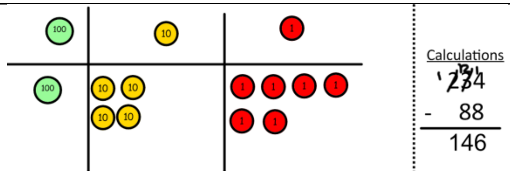
Find the difference (numbers beyond 20) (Counting up is often easier than back. Please stress the use of the word 'difference' as it is often misunderstood)	Compare amounts and objects to find the difference. Use Numicon or Dienes to make bars to find the difference. Counting up would be taught as a more efficient strategy by bridging though ten or doing bigger jumps of tens then ones	 Count on to find the difference. 	Hannah has 27 sandwiches, Helen has 15 sandwiches. What is the difference between them?
Subtraction by partitioning **(only partition the second number) Useful for more mental and informal strategies supported with jottings**	**Subtracting 10s and 1s** For example: 18 – 12 Subtract 12 by first subtracting the 10, then the remaining 2.  *First subtract the 10, then take away 2.*	**Subtracting 10s and 1s** For example: 18 – 12 Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.* Start at the bigger number and count back the smaller number showing the jumps on the number line or hundreds square. 57 - 23	**Subtracting 10s and 1s** Use a part-whole model to support the calculation. 57 – 23 = 57 - 20 - 3

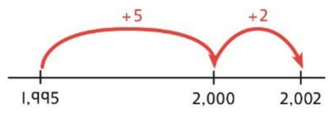
			
Column method without exchange (remember to include 0s) Although WR doesn't move to this we will as an efficient method for calculating with 2 digit numbers	75 - 42 ONLY MAKE THE BIGGER NUMBER Use Base 10 to make the bigger number (only) then take the smaller number away. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working. Cross out what is removed 	This will lead to a clear  written column subtraction.
GD If capable, moving to formal column (2 digit) with exchange – see below		May draw B10 and cross out a ten and draw in ten ones to show exchange	

Subtraction in Years 3 – 6 with increasingly large numbers

Objective and Strategies	Concrete	Pictorial	Abstract
3 digit no exchange ONLY MAKE THE BIGGER NUMBER	 H T O 9 9 9 – 3 5 2 7 H T O 9 9 9 – 3 5 2 4 7 H T O 9 9 9 – 3 5 2 6 4 7	 	H T O 9 9 9 – 3 5 2 7 H T O 9 9 9 – 3 5 2 4 7 H T O 9 9 9 – 3 5 2 6 4 7
Column method with exchange (remember to include 0) (Year 3 , three digit Year 4, four digit etc)	**Use Base 10 to start with before moving on to place value counters**. Start with only one exchange before moving onto subtraction with 2 or more .  	Draw Base Ten blocks. Cross out what is taken away.Show exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 

	Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  	Missing digit reasoning with increasing complexity. 

<table>
<tr>
<td></td>
<td colspan="2">

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Choosing efficient methods</td>
<td colspan="2"></td>
<td colspan="2">To subtract two large numbers that are close, children find the difference by counting on.
2,002 − 1,995 = ?


Use addition to check subtractions.
I calculated 7,546 − 2,355 = 5,191.
I will check using the inverse.</td>
</tr>
<tr>
<td>Subtracting decimals Y4 –Y6</td>
<td>Explore complements to a whole number e.g. by working in the context of length.


1 m − ☐ m = ☐ m
1 − 0·49 = ?</td>
<td colspan="2">Use a place value grid to represent the stages of column subtraction, including exchange where required.
5·74 − 2·25 = ?</td>
<td>Use column subtraction, with an understanding of place value, including subtracting numbers *with different numbers of decimal places*.
3·921 − 3·75 = ?
<table>
<tr><th></th><th>O</th><th>·</th><th>Tth</th><th>Hth</th><th>Thth</th></tr>
<tr><td></td><td>3</td><td>·</td><td>9</td><td>2</td><td>1</td></tr>
<tr><td>−</td><td>3</td><td>·</td><td>7</td><td>5</td><td>0</td></tr>
<tr><td></td><td></td><td>·</td><td></td><td></td><td></td></tr>
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<table>
<tr>
<td></td>
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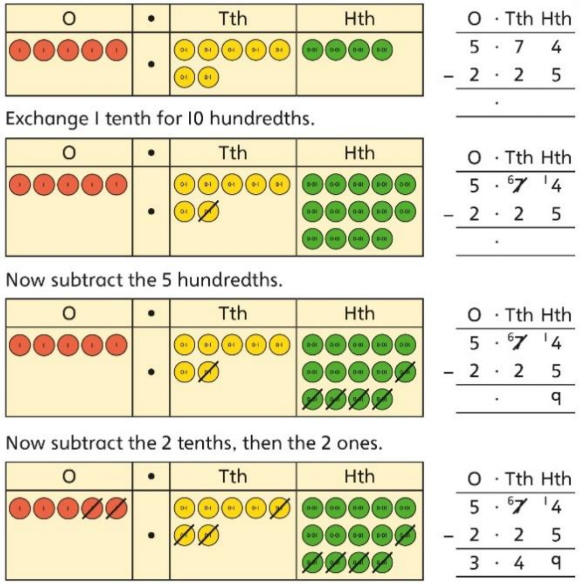
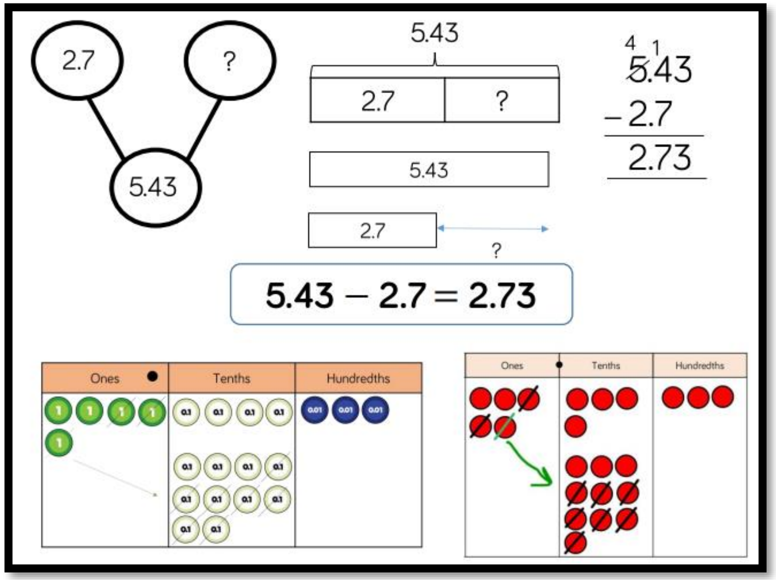
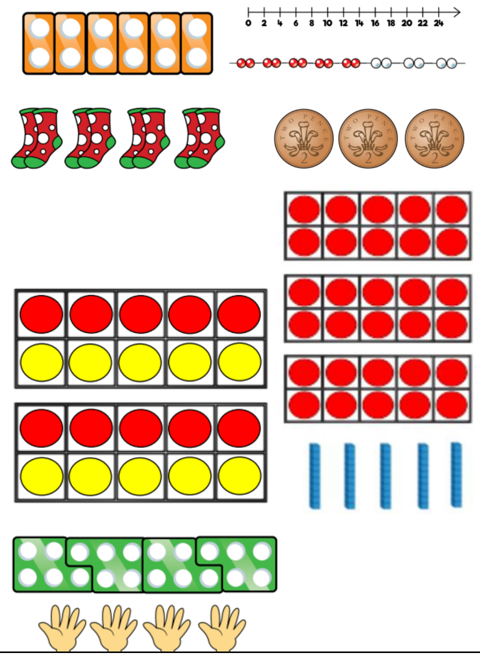
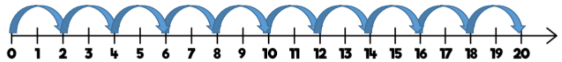
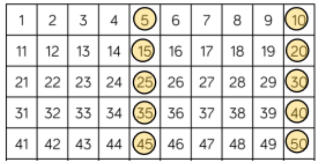
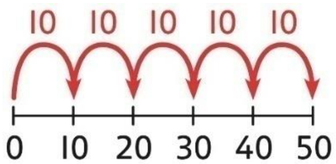
<td>
</td>
<td>$$\begin{array}{rrrcr} & {}^{5} & {}^{12} & & {}^{1} \\ 2 & \cancel{6} & \cancel{3} & . & 0 \\ - & 2 & 6 & . & 5 \\ \hline 2 & 3 & 6 & . & 5 \end{array}$$</td>
</tr>
</table>

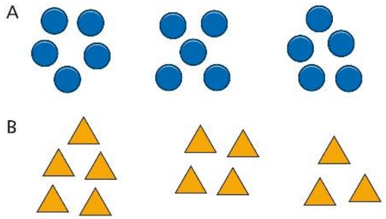
Bar models, cherry part/whole and representations with counters continue to be used to model calculations in Y4-6 with integers and decimals.



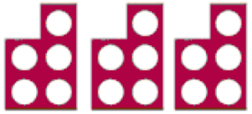
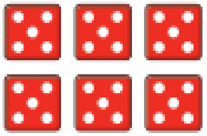
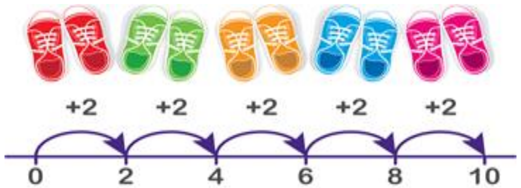
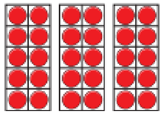
Multiplication

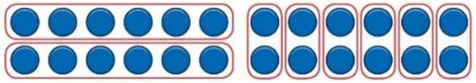
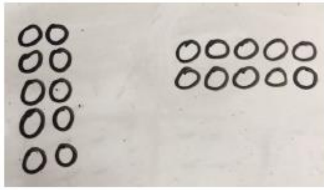
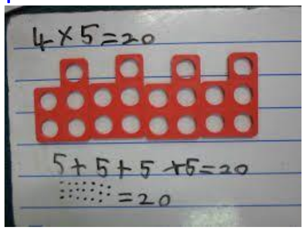
Year 1

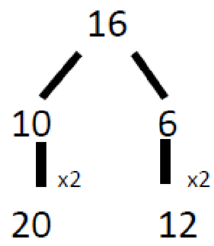
Objective and Strategies	Concrete	Pictorial	Abstract
Counting in 2s , 5s and 10s		**Use numberlines and grids**  	Opportunities for orally skip counting forwards and backwards, writing down and looking for patterns Finding the total of equal groups by counting in 2s, 5s and 10s 

Recognising equal groups	**Recognising and making equal groups** Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.  	**Recognising and making equal groups** Children draw and represent equal and unequal groups.  	**Describe equal groups using words** 'there are ___ groups of ___.'
Doubling	Use practical activities to show how to double a number. Doubles ona ten frame and Rekenrek	Draw pictures to show how to double a number. Double 4 is 8 	2 groups of 4 = 8 4 + 4 = 8

Multiplication in Year 2

Objective and Strategies	Concrete	Pictorial	Abstract
Repeated addition moving to multiplication sy,bol	 5 + 5 + 5 =   5 jumps of 2 on a number line	 _____ + _____ + _____ = _____ x _____	10 + 10 + 10 = 3 groups of 10 3 x 10 = 5+ 5+ 5+ 5+ 5+ 5 = 6 groups of 5 6 x 5 =

Arrays- showing commutative multiplication (2s, 5s and 10s only)	Create arrays using counters/ cubes to show multiplication sentences.  4 groups of 5	Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.  *This is 2 groups of 6 and also 6 groups of 2.* Children to represent the arrays pictorially. 	Use an array to write multiplication sentences and reinforce repeated addition. '3 rows of 5' '5 columns of 3'  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Learning ×2, ×5 and ×10 table facts	Build with Numicon/ other and notice patterns in answers.  		Understand how the times-tables increase and contain patterns

Doubling in year 2			Partition a number and then double each part before recombining it back together 

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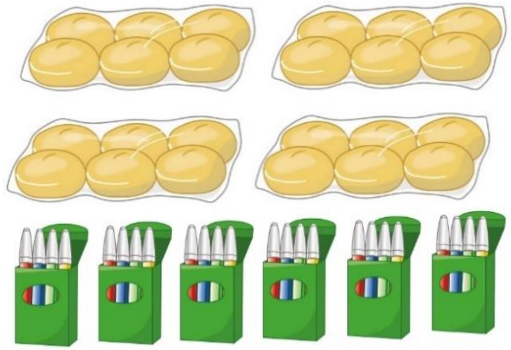
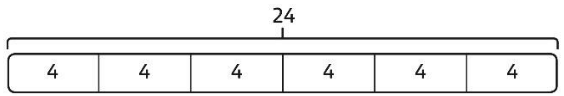
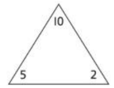
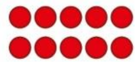
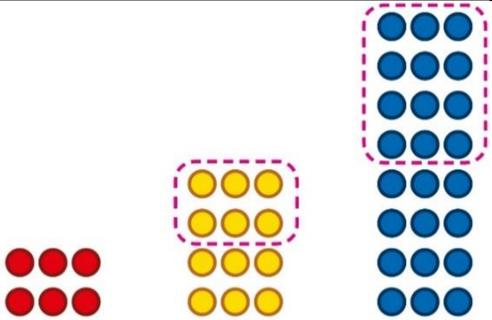
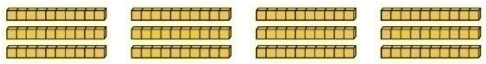
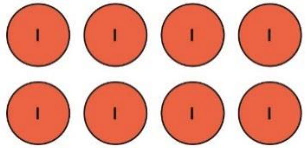
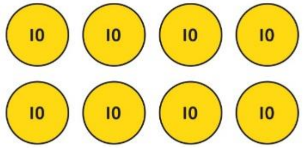
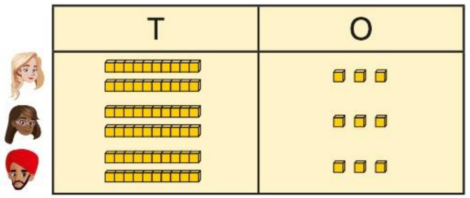
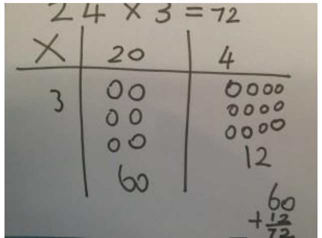
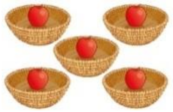
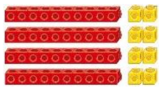
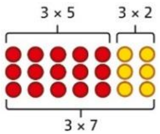
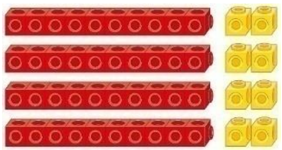
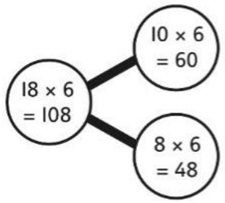
Multiplication in Year 3

Objective and Strategies	Concrete	Pictorial	Abstract
Using commutativity to support understanding of the times-tables 3s, 4s, 8s *There are 6 groups of 4 pens.*	*There are 4 groups of 6 bread rolls.* *I can use 6 × 4 = 24 to work out both totals.* 	A bar model may represent multiplications as equal groups. Understand how times-table facts relate to commutativity 	3 x 4 = 4 x 3  *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*
Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling	 3 × 2 = 6 3 × 4 = 12 3 × 8 = 24	.	
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment.	Understand how unitising 10s supports multiplying by multiples of 10.	*Once patterns are noticed then children can use place value sliders to mover digits and add a placeholder zero at the end.*

	Make 4 groups of 3 ones.  *Make 4 groups of 3 tens.*  *What is the same?* *What is different?*	  *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	*4 x 3 = 12* *Then* *4 x 30 = 120*
Multiplying a 2-digit number by a 1-digit number **(informal partitioning method used to support mental methods and <u>only for x1 digit number</u>. Recorded on paper but children will move on from this method in y4)**	**Understand how to link partitioning a 2-digit number with multiplying.** *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.* *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	*Could draw place value counters or B10 if needed*  *Use addition to complete multiplications of 2-digit numbers by a 1-digit number*	*3 × 24 =* *3 × 4 = 12 and 3 × 20 = 60* *12+ 60 = 72* *So 3 x 24 = 72* Used to be known as grid method. **Children often find drawing the actual grid troublesome so just recording partial products. (distributive law) as the expanded method is preferable** 24 <u>X 3</u> 12 (3x4) <u>60</u> (3 x 20) 72

Multiplication in Year 4-6

Under-Standing times-tables up to 12 × 12	Understand the special cases of multiplying by **1 and 0.**   *5 × 1 = 5* *5 × 0 = 0* Look for patterns e.g. in 9s Use what you know - commutativity Represent the ×12 tables in relation to the ×10 table.(use partitioning)  *4 × 12 = 40 + 8*	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5× 6 is double 5× 3* ×5 table and ×6 table *I know that 7× 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  Do not put too much emphasis upon this factorisation. Work more towards fluency of factual recall. ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 − 6*
Understanding and using partitioning in multiplication for mental methods	Make multiplications by partitioning. *4 × 12 is 4 groups of 10 and 4 groups of 2.* 	Use partitioning to multiply 2-digit numbers by a single digit for mental methods. *18 × 6 = ?*  $18 \times 6 = \underbrace{10 \times 6}_{= 60} + \underbrace{8 \times 6}_{48}$ $= 108$

4 × 12 = 40 + 8

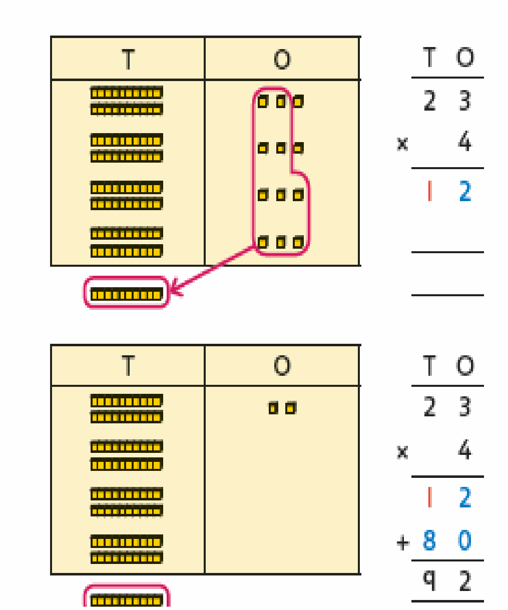
18 × 6 = 10 × 6 + 8 × 6 = 60 + 48 = 108

Formal short and long multiplication

Whilst partitioning with jottings is fine for mental methods, written formal methods are used (formal short then long multiplication)

An efficient method for larger numbers.

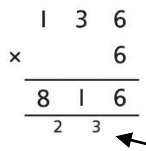
Y3 2 digit x 1
Y4 3 digit x 1
Y5 4 digit x 1
then 2d x 2 , 3 d x 2 , 4 d x 2



Make the link between the partioned expanded method and the more efficient short method.Show how the ten is regrouped.

Start with expanded multiplication, reminding the children about lining up their numbers clearly in columns.**This is only *used initially to explain the more compact way* of recording using short and long multiplication**.

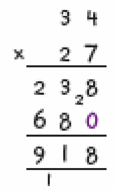
The goal is the formal short and long multiplication methods.



1241 x 3 = 3723

$$\begin{array}{r} 1241 \\ \times \quad 3 \\ \hline 3723 \\ \hline {\scriptstyle 1} \end{array}$$

Carried digits are placed below the line for short multiplication.



	Th	H	T	O
		2	3	4
×			3	2
		4	6	8
	$_1$7	$_1$0	2	0
	7	4	8	8

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 $_2$	1 $_5$	9 $_3$	1 $_7$	2
5 $_1$	4	7 $_1$	8	0
7	6	6	9	2
		1		

NOTE: Any carried digits are placed neatly next to the digit in the product part of the calculation.

Application: Missing digit from year 3 and decimal multiplication from year 5 to follow.

<table>
<tr>
<td>Y4
Multiplying more than two numbers</td>
<td>Represent situations by multiplying three numbers together.

Each sheet has 2 × 5 stickers.There are 3 sheets.

There are 5 × 2 × 3 stickers in total.

$\underbrace{5 \times 2}_{10} \times 3 = 30$
$10 \times 3 = 30$</td>
<td>12 x 2 x 5
Do 2 x 5 first = 10
Then 12 x 10 -120

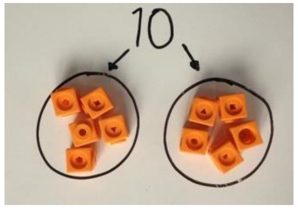
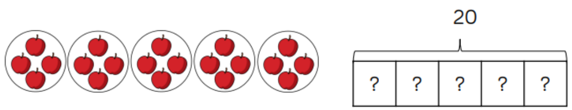
Again, don’t spend too long as this is more for efficient mental methods. Children are required to know formal short method.</td>
</tr>
<tr>
<td>Multiplying decimals by 10,100 and 1,000</td>
<td>Use place value equipment to explore and understand the of 10 tenths, 10 hundredths or 10 thousandths

Use PV sliders</td>
<td>Understand how this is represented on a place value chart.
<table>
<tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th><th>•</th><th>Tth</th></tr>
<tr><td></td><td></td><td></td><td></td><td>2</td><td>•</td><td>5</td></tr>
<tr><td>2·5 × 10 = 25</td><td></td><td></td><td>2</td><td>5</td><td>•</td><td></td></tr>
<tr><td>2·5 × 100 = 250</td><td></td><td>2</td><td>5</td><td>0</td><td>•</td><td></td></tr>
<tr><td>2·5 × 1,000 = 2,500</td><td>2</td><td>5</td><td>0</td><td>0</td><td>•</td><td></td></tr>
</table></td>
</tr>
<tr>
<td>Multiply one-digit numbers with up to two decimal places by integers</td>
<td colspan="2">Use formal multiplication method but ask children to make an estimation first so that decimal is placed in correct place e.g.

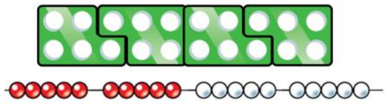
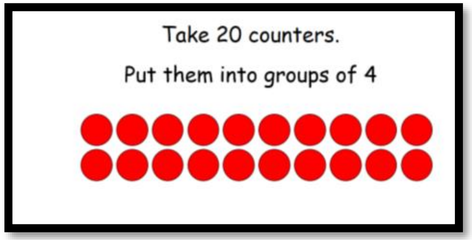
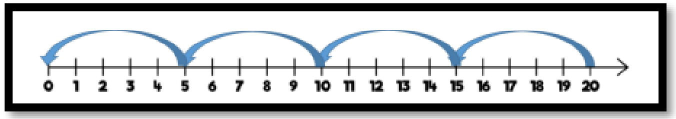
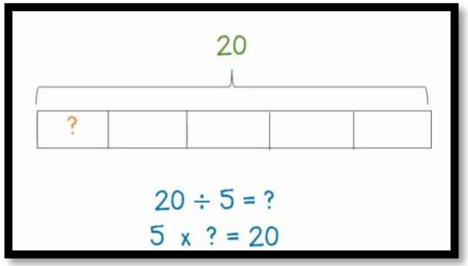
2.35 x 4 would round to 2 x 4 = 8 so answer should be a little more than 8.

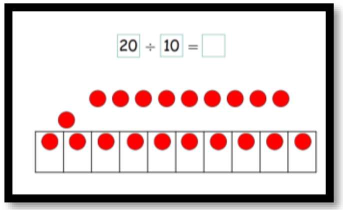
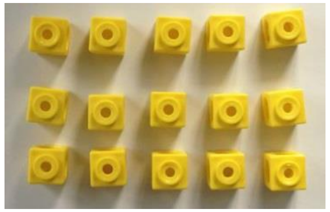
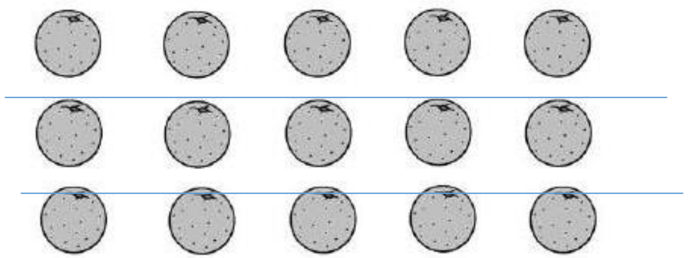
Remove decimal from calculation and place back in the product so answer is a little more than 8.
2 3 5
X 4
9 4 0 so 9.40
1 2
Also use language of ‘If I make the number 100 times larger my answer will be 100 times larger so I need to make it one hundredth of the size by placing the decimal with 2dp’</td>
</tr>
</table>

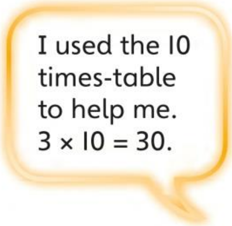
<u>Division in Year 1</u>

Objective and Strategies	Concrete	Pictorial	Abstract
Equal grouping Children arrange things in equal groups	Start with a pile of 20 cubes Put into groups of 5	 Draw circles around grops of five Draw groups of 5 Put 5s in a bar model	No formal use of division sign
Sharing objects <u>into equal groups</u>	 I have 10 cubes, can you share them equally in 2 groups?  **Make link with half.**	Children use pictures or shapes to share quantities.  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?	20 bananas shared between 5 boxes is ?

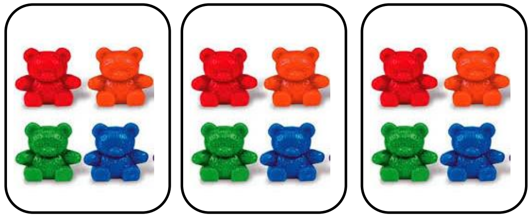
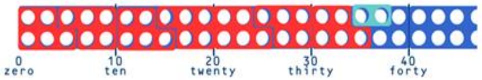
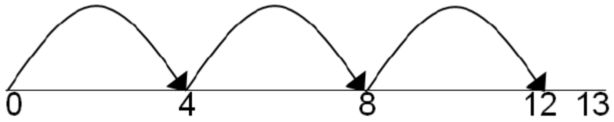
<u>Division in Year 2</u>

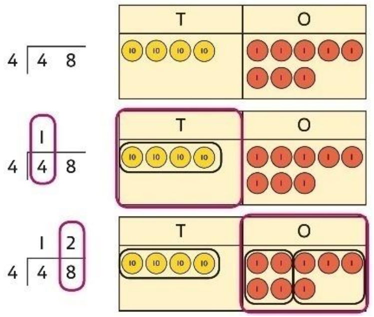
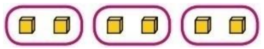
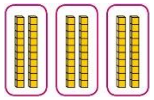
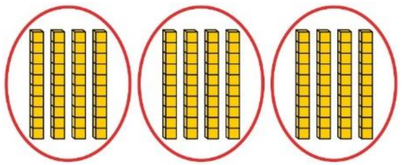
Division <u>as grouping</u> Important that children are exposed to sharing and grouping aspects of division Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division	Place Numicon 5s on to 20 and count how many groups of 5.   	Using their knowledge of the inverse relationship between multiplication and division, children can use their multiplication tables when grouping on a number line, working from left to right.  How many groups of 5 are there in 20? Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	Division sign is introduced $20 \div 5 = 4$

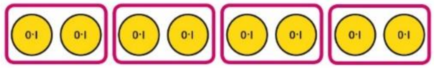
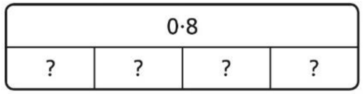
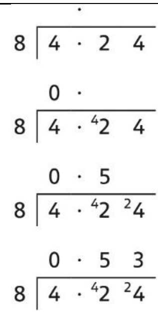
Divison as sharing			
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Using known times-tables to solve divisions	$1 \times 10 = 10$ $2 \times 10 = 20$ **$3 \times 10 = 30$** $4 \times 10 = 40$ $5 \times 10 = 50$ $6 \times 10 = 60$ $7 \times 10 = 70$ $8 \times 10 = 80$  Relate times-table knowledge directly to division *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *$3 \times 10 = 30$ so $30 \div 10 = 3$*

Division in Year 3 - 6

Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over   Use Numicon. Make 37 then place 5s over to see how many 5s and what remains.  How many 5’s in 37?	 Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using **r**.  29 ÷ 8 = 3 r 5 **Also understand that the remainder cannot be equal to or larger than the groups you are making**.

Short division 'bus stop' ***'Stand alone' method → for efficiency*** Year 3 two digit by 1 digit Year 4 three digit by 1 digit Year 5 four digit by 1 digit Year 6 four digit by 2 digit	Place value counters *could* be used if teacher deems necessary for understanding and it helps children understand the ***Grouping*** method used (rather than the PV aspect). However, this concrete method can cause confusion and often best to move straight to abstract. 	Children represent the process with drawing the counters and finding groups. Common misconception is not understanding what digit is carried as a remainder when the first digit cannot be divided by the divisor.	**Begin with divisions that divide equally with no remainder.** Move onto divisions with a remainder. 124 3 | 37^{1}2 0 5 5 6 7 | 3 ^{3}8 ^{3}9 ^{4}2 Use multiplication to check. *556 × 7 = ?* **Remember that children tend to make mistakes when zero is involved. Ensure this is addressed.**
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. *Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	Divide multiples of 10 by unitising.  *12 tens shared into 3 equal groups.* *4 tens in each group.*	Divide multiples of 10 by a single digit using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*

			Y5/6 Find families of facts from a known multiplication. *I know that 18 × 4 = 72.* *This can help me work out:* *1·8 × 4 = ?* *18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?*
Dividing decimals Y6 division of decimal numbers by one-digit integer	 Use place value equipment to explore division of decimals. *8 tenths divided into 4 groups. 2 tenths in each group.*	 4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	 Use short division to divide decimals with up to 2 decimal places. Remind students to place the decimal in the answer directly above the dividend.

Two digit divisors AT CTK We use bus stop for two digit divisors with children supporting this method with listing multiples beyond 12s	see below

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

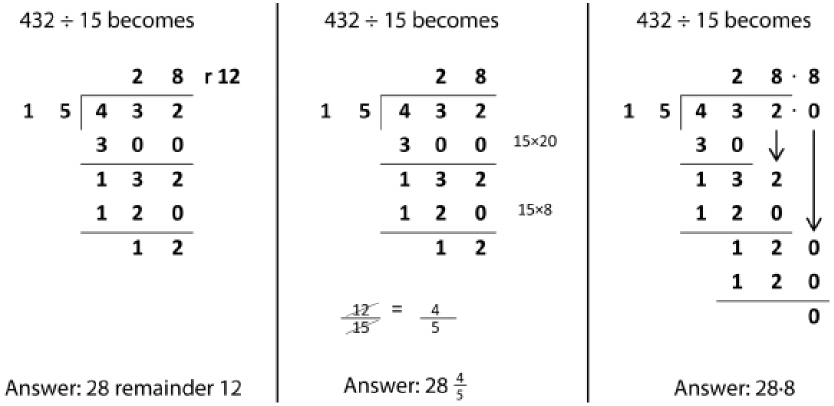
	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

Express remainders as a fraction and decimal

Long Division
Y6 formal written method
(decision not to use at CTK)

Long division



National Curriculum shows the formal 'bring down' method. It also shows how remainders can be expressed as r, fractions and decimals. Children need to be able to make sense of the remainder, including recognising when to use a decimal esp. money and measure.

Common misconception: children use the remainder and just put it after a decimal.

Glossary for Addition and Subtraction

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

Glossary for Multiplication and Division

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor